Whole Numbers

1

SECTION 1.2

Addition with Whole Numbers, and Perimeter

Objectives

A Add whole numbers.

B Understand the notation and vocabulary of addition.

C Use the properties of addition.

D Find a solution to an equation by inspection.

E Find the perimeter of a figure.

Facts of Addition

Facts of Addition

Using lengths to visualize addition can be very helpful. In mathematics we generally do so by using the number line.

For example, we add 3 and 5 on the number line like this: Start at 0 and move to 3, as shown in Figure 1.

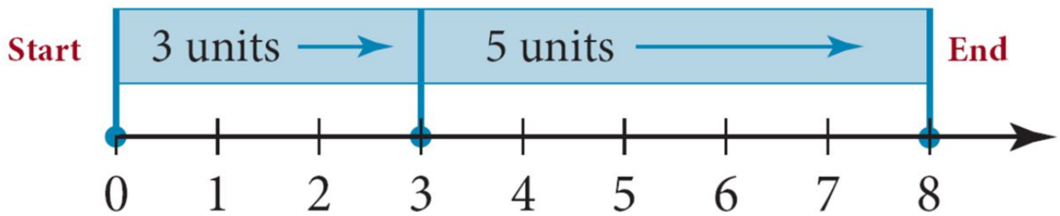

Figure 1

From 3, move 5 more units to the right. This brings us to 8. Therefore, 3 + 5 = 8.

Facts of Addition

If we do this kind of addition on the number line with all combinations of the numbers 0 through 9, we get the results summarized in Table 1. We call the information in Table 1 our basic addition facts.

TABLE 1

ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Facts of Addition

Your success with the examples and problems in this section depends on knowing the basic addition facts.

Suppose we want to use the table to find the answer to 3 + 5. We locate the 3 in the column on the left and the 5 in the row at the top.

We read *across* from the 3 and *down* from the 5. The entry in the table that is across from 3 and below 5 is 8.

Therefore, 3 + 5 = 8.

A Adding Whole Numbers

Adding Whole Numbers

Now let's use the addition facts to help us add whole numbers with more than one digit.

To add whole numbers, we add digits within the same place value.

First, we add the digits in the ones place, then the tens place, then the hundreds place, and so on.

Example 1

Add: 43 + 52

Solution:

This type of addition is best done vertically. First we add the digits in the ones place.

$$\begin{array}{r} 4\mathbf{3} \\ +\ 5\mathbf{2} \\ \hline 5 \end{array}$$

Then we add the digits in the tens place.

$$\begin{array}{r} 43 \\ +\ 52 \\ \hline 95 \end{array}$$

Addition with Carrying

Addition with Carrying

In Example 1, the sums of the digits with the same place value was always 9 or less.

There are many times when the sum of the digits with the same place value will be a number larger than 9.

In these cases we have to do what is called *carrying* in addition. The following examples illustrate this process.

Example 3

Add: 197 + 213 + 324

Solution:

We write the sum vertically and add digits with the same place value.

$$\begin{array}{r} \overset{1}{1}97 \\ 213 \\ +\ \ 324 \\ \hline 4 \end{array}$$

When we add the ones, we get $7 + 3 + 4 = 14$.
We write the 4 and carry the 1 to the tens column.

Example 3 – *Solution*

cont'd

$$\begin{array}{r} {}^{1\,1} \\ 197 \\ 213 \\ +\ \ 324 \\ \hline 34 \end{array}$$

We add the tens, including the 1 that was carried over from the last step. We get 13, so we write the 3 and carry the 1 to the hundreds column.

$$\begin{array}{r} {}^{1\,1} \\ 197 \\ 213 \\ +\ \ 324 \\ \hline 734 \end{array}$$

We add the hundreds, including the 1 that was carried over from the last step, and write the total.

B Vocabulary

Vocabulary

The word we use to indicate addition is the word *sum*. If we say “the sum of 3 and 5 is 8,” what we mean is $3 + 5 = 8$.

The word sum always indicates addition. We can state this fact in symbols by using the letters *a* and *b* to represent numbers in the following definition.

Definition

If a and b are any two numbers, then the **sum** of a and b is $a + b$. To find the sum of two numbers, we add them.

Vocabulary

Table 2 gives some phrases and sentences in English and their mathematical equivalents written in symbols.

TABLE 2

In English	In Symbols
The sum of 4 and 1	$4 + 1$
4 added to 1	$1 + 4$
8 more than m	$m + 8$
x increased by 5	$x + 5$
The sum of x and y	$x + y$
The sum of 2 and 4 is 6.	$2 + 4 = 6$

C Properties of Addition

Properties of Addition

Once we become familiar with addition, we may notice some facts about addition that are true regardless of the numbers involved.

The first of these facts involves the number 0 (zero). Whenever we add 0 to a number, the result is the original number.

Properties of Addition

For example,

$$7 + 0 = 7 \qquad \text{and} \qquad 0 + 3 = 3$$

Because this fact is true no matter what number we add to 0, we call it a property of 0.

Addition Property of 0

If we let a represent any number, then it is always true that

$$a + 0 = a \qquad \text{and} \qquad 0 + a = a$$

In words: Adding 0 to any number leaves that number unchanged.

Properties of Addition

A second property we notice by becoming familiar with addition is that the order of two numbers in a sum can be changed without changing the result.

$$3 + 5 = 8 \quad \text{and} \quad 5 + 3 = 8$$

$$4 + 9 = 13 \quad \text{and} \quad 9 + 4 = 13$$

This fact about addition is true for *all* numbers. The order in which you add two numbers doesn't affect the result.

Properties of Addition

We call this fact the *commutative property of addition*, and we write it in symbols as follows.

Commutative Property of Addition

If a and b are any two numbers, then it is always true that

$$a + b = b + a$$

In words: Changing the order of two numbers in a sum doesn't change the result.

Example 5

Use the commutative property of addition to rewrite each sum.

a. 4 + 6 **b.** 5 + 9 **c.** 3 + 0 **d.** 7 + *n*

Solution:

The commutative property of addition indicates that we can change the order of the numbers in a sum without changing the result.

Applying this property we have:

a. 4 + 6 = 6 + 4

Example 5 – *Solution*

cont'd

b. $5 + 9 = 9 + 5$

c. $3 + 0 = 0 + 3$

d. $7 + n = n + 7$

Notice that we did not actually add any of the numbers.

The instructions were to use the commutative property, and the commutative property involves only the order of the numbers in a sum.

Properties of Addition

The last property of addition we will consider here has to do with sums of more than two numbers. Suppose we want to find the sum of 2, 3, and 4. We could add 2 and 3 first, and then add 4.

$$(2 + 3) + 4 = 5 + 4 = 9$$

Or, we could add the 3 and 4 together first and then add the 2.

$$2 + (3 + 4) = 2 + 7 = 9$$

The result in both cases is the same.

Properties of Addition

If we try this with any other numbers, the same thing happens.

We call this fact about addition the *associative property of addition*, and we write it in symbols as follows:

Associative Property of Addition

If a, b, and c represent any three numbers, then

$$(a + b) + c = a + (b + c)$$

In words: Changing the grouping of three numbers in a sum doesn't change the result.

Properties of Addition

The commutative and associative properties of addition tell us that when adding whole numbers, we can use any order and grouping.

When adding several numbers, it is sometimes easier to look for pairs of numbers whose sums are 10, 20, and so on.

D Solving Equations

Solving Equations

We can use the addition table to help solve some simple equations. If *n* is used to represent a number, then n is called a *variable*. The equation

$$n + 3 = 5$$

will be true if *n* is 2.

The number 2 is therefore called a *solution* to the equation, because, when we replace *n* with 2, the equation becomes a true statement:

$$2 + 3 = 5$$

Solving Equations

Equations like this are really just puzzles, or questions. When we say, “Solve the equation $n + 3 = 5$,” we are asking the question, “What number do we add to 3 to get 5?”

When we solve equations by reading the equation to ourselves and then stating the solution, as we did with the equation above, we are solving the equation by *inspection*.

Example 8

Find the solution to each equation by inspection.

a. $n + 5 = 9$

b. $n + 6 = 12$

c. $4 + n = 5$

d. $13 = n + 8$

Example 8 – *Solution*

We find the solution to each equation by using the addition facts given in Table 1.

TABLE 1

ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9
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1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Example 8 – *Solution*

cont'd

a. The solution to $n + 5 = 9$ is 4, because $4 + 5 = 9$.

b. The solution to $n + 6 = 12$ is 6, because $6 + 6 = 12$.

c. The solution to $4 + n = 5$ is 1, because $4 + 1 = 5$.

d. The solution to $13 = n + 8$ is 5, because $13 = 5 + 8$.

E Perimeter

Perimeter

FACTS FROM GEOMETRY **Perimeter**

Here we will look at several different shapes called *polygons*. A *polygon* is a closed geometric figure, with at least three sides, in which each side is a straight line segment.

The most common polygons are squares, rectangles, and triangles. Examples of these are shown in Figure 2.

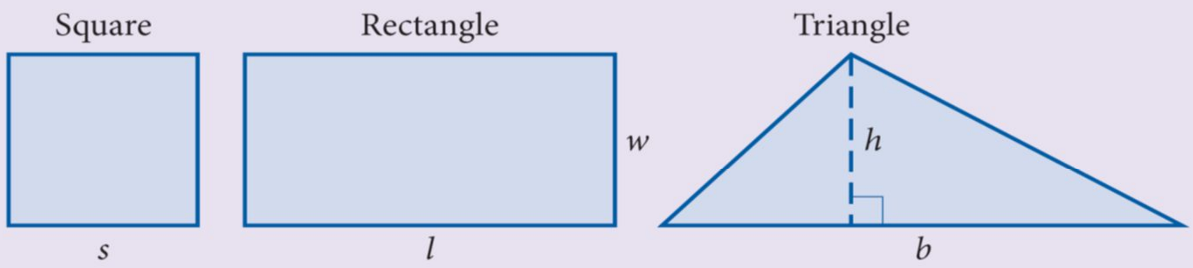


FIGURE 2

In the square, s is the length of the side, and each side has the same length. In the rectangle, l stands for the length, and w stands for the width. The width is usually the lesser of the two. The b and h in the triangle are the base and height, respectively. The height is always perpendicular to the base. That is, the height and base form a 90°, or *right*, angle where they meet.

To find the perimeter of a polygon, we add the lengths of all the sides. For example, the perimeter of the square in Figure 2 would be $s + s + s + s$.

Perimeter

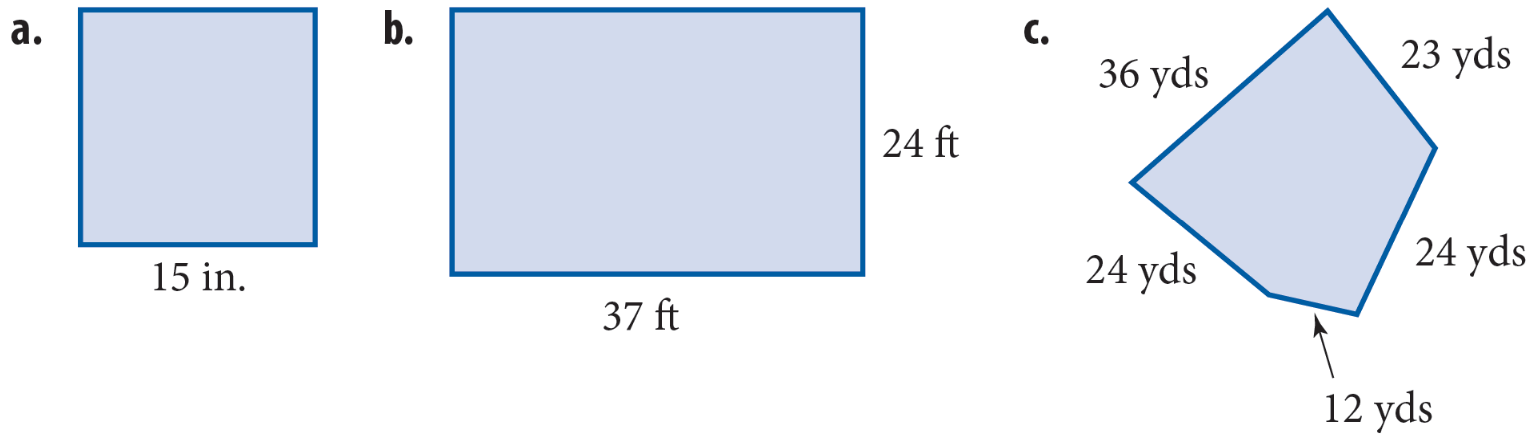
Here is a formal definition to perimeter:

Definition

The **perimeter** of any polygon is the sum of the lengths of the sides, and it is denoted with the letter P.

Example 9

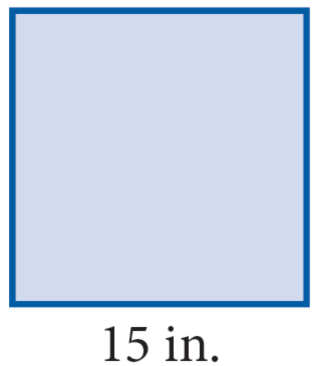
Find the perimeter of each geometric figure. Figure a is a square.



Example 9(a) – *Solution*

In each case, we find the perimeter by adding the lengths of all the sides.

a.



The figure is a square. Because the length of each side in the square is the same, the perimeter is

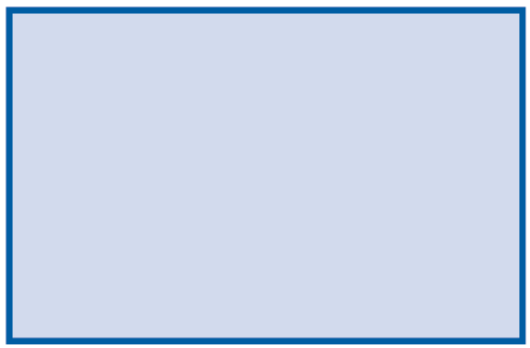
$$P = 15 + 15 + 15 + 15$$

$$= 60 \text{ inches}$$

Example 9(b) – *Solution*

cont'd

In the rectangle, two of the sides are 24 feet long, and the other two are 37 feet long.



The perimeter is the sum of the lengths of the sides.

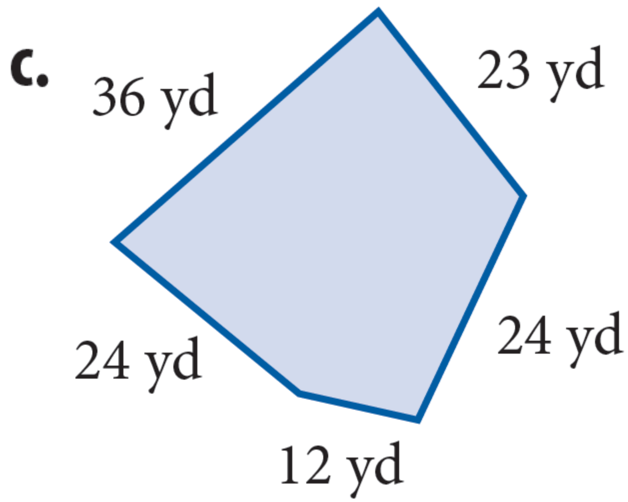
$$P = 24 + 24 + 37 + 37$$

$$= 122 \text{ feet}$$

Example 9(c) – *Solution*

cont’d

For this polygon, we add the lengths of the side together.



The result is the perimeter.

$$P = 36 + 23 + 24 + 12 + 24$$

$$= 119 \text{ yards}$$