Chapter 2

# Parallel Lines

# 2.5 Convex Polygons

# Convex Polygons

### Definition

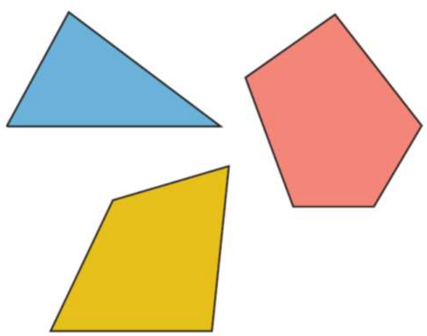
A **polygon** is a closed plane figure whose sides are line segments that intersect only at the endpoints.

The polygons we generally consider those are **convex**; the angle measures of convex polygons are between 0° and 180°.

# Convex Polygons

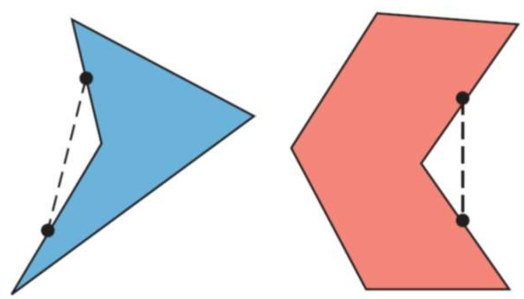
Convex polygons are shown in Figure 2.28; those in Figure 2.29 are **concave**. A line segment joining two points of a concave polygon can contain points in the exterior of the polygon.

Thus, a concave polygon always has at least one reflex angle.



Convex Polygons

**Figure 2.28**

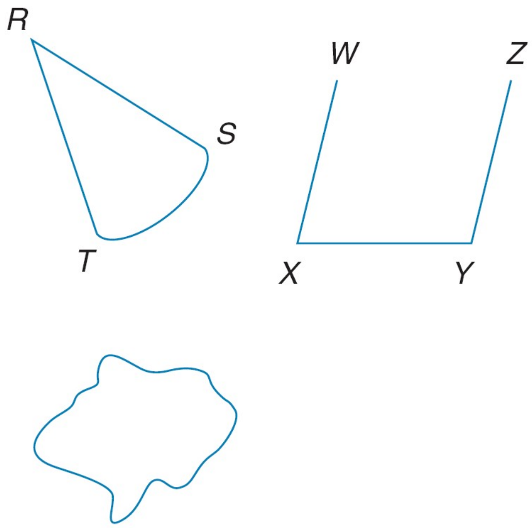


Concave Polygons

**Figure 2.29**

# Convex Polygons

Figure 2.30 shows some figures that aren’t polygons at all!




Not Polygons

**Figure 2.30**

# Convex Polygons

Table 2.3 categorizes polygons by their number of sides.

**TABLE 2.3**

| Polygon | Number of Sides | Polygon | Number of Sides |
|---|---|---|---|
| Triangle | 3 | Heptagon | 7 |
| Quadrilateral | 4 | Octagon | 8 |
| Pentagon | 5 | Nonagon | 9 |
| Hexagon | 6 | Decagon | 10 |

With Venn Diagrams, the set of all objects under consideration is called the **universe**.

# Convex Polygons

If $P$ = {all polygons} is the universe, then we can describe sets $T$ = {triangles} and $Q$ = {quadrilaterals} as subsets that lie within universe $P$.

Sets $T$ and $Q$ are described as **disjoint** because they have no elements in common. See Figure 2.31.

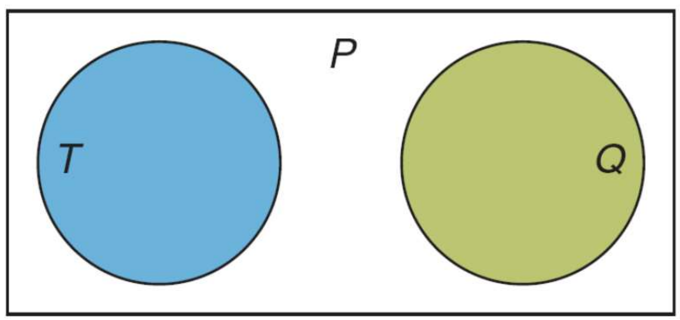



Figure 2.31

# DIAGONALS OF A POLYGON

# Diagonals of a Polygon

A **diagonal** of a polygon is a line segment that joins two nonconsecutive vertices.

Figure 2.32 shows heptagon *ABCDEFG* for which $\angle$ GAB, $\angle$B, and $\angle$BCD are some of the interior angles and $\angle$1, $\angle$2, and $\angle$3 are some of the exterior angles. $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are some of the sides of the heptagon, because these join consecutive vertices.

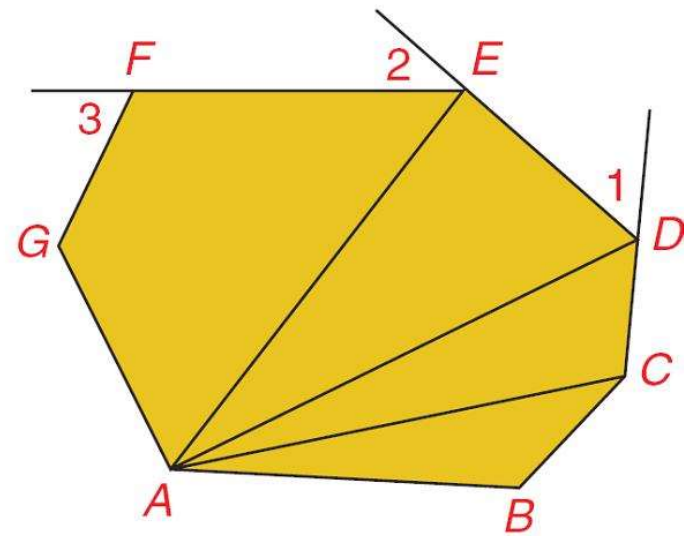
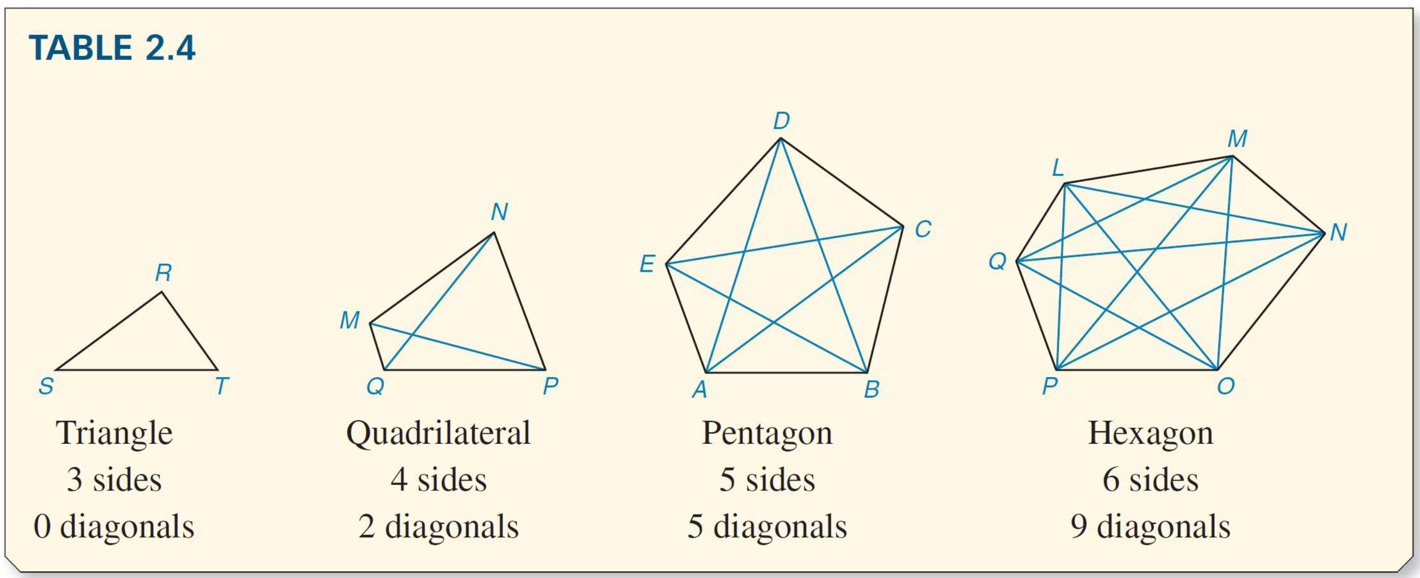


Figure 2.32

# Diagonals of a Polygon

Because a diagonal joins nonconsecutive vertices of *ABCDEFG*, $\overline{AC}$, $\overline{AD}$, and $\overline{AE}$ are among the many diagonals of the polygon.

Table 2.4 illustrates polygons by numbers of sides and the corresponding total number of diagonals for each type.



# Diagonals of a Polygon

When the number of sides of a polygon is small, we can list all diagonals by name.

For pentagon *ABCDE* of Table 2.4, we see diagonals $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$, and $\overline{CE}$—a total of five.

As the number of sides increases, it becomes more difficult to count all the diagonals.

# Diagonals of a Polygon

Theorem 2.5.1

The total number of diagonals $D$ in a polygon of $n$ sides is given by the formula

$$D = \frac{n(n - 3)}{2}.$$

Theorem 2.5.1 reaffirms the fact that a triangle has no diagonals; when $n = 3$,

$$D = \frac{3(3 - 3)}{2} = 0.$$

# Example 1

Find (a) the number of diagonals for any pentagon (b) the type of polygon that has 9 diagonals.

Solution:

(a) For a pentagon, $n = 5$.

Then

$$D = \frac{5(5 - 3)}{2} = \frac{5(2)}{2} = 5.$$

Thus, the pentagon has 5 diagonals.

# Example 1 – *Solution*

cont'd

(b)

$$\frac{n(n-3)}{2} = 9$$

$$\frac{n^2 - 3n}{2} = 9$$

$$n^2 - 3n = 18$$

$$n^2 - 3n - 18 = 18$$

$$(n - 6)(n + 3) = 0$$

$$n - 6 = 0 \quad \text{or} \quad n + 3 = 0$$

$$n = 6 \quad \text{or} \quad n = -3 \text{ (discard)}$$

When $n = 6$, the polygon is a hexagon.

# SUM OF THE INTERIOR ANGLES OF A POLYGON

# Sum of the Interior Angles of a Polygon

The following theorem provides the formula for the sum of the interior angles of any polygon.

Theorem 2.5.2

The sum $S$ of the measures of the interior angles of a polygon with $n$ sides is given by $S = (n - 2) \cdot 180°$. Note that $n > 2$ for any polygon.

# Example 2

Find the sum of the measures of the interior angles of a hexagon. Then find the measure of each interior angle of an equiangular hexagon.

Solution:

For the hexagon, $n = 6$, so the sum of the measures of the interior angles is $S = (6 - 2) \cdot 180°$ or $4(180°)$ or $720°$.

In an equiangular hexagon, each of the six interior angles measures $\frac{720°}{6}$, or $120°$.

# REGULAR POLYGONS

# Regular Polygons

Figure 2.34 shows polygons that are, respectively, (a) **equilateral,** (b) **equiangular,** and (c) **regular** (both sides and angles are congruent). Note the dashes that indicate congruent sides and the arcs that indicate congruent angles.

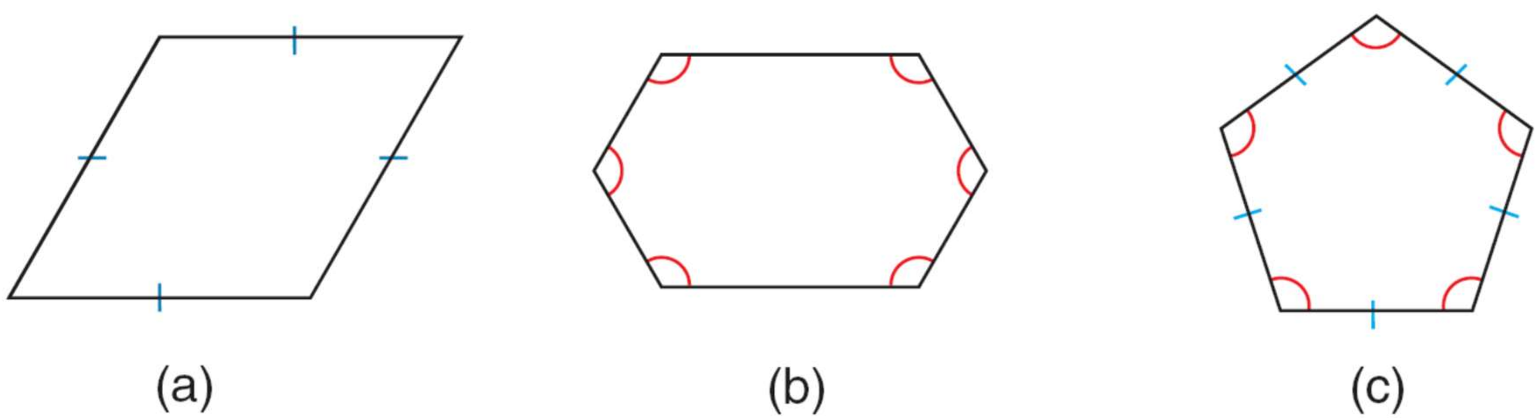



**Figure 2.34**

# Regular Polygons

Definition

A **regular polygon** is a polygon that is both equilateral and equiangular.

Corollary 2.5.3

The measure *I* of each interior angle of a regular polygon or equiangular polygon of *n* sides is

$$I = \frac{(n - 2) \cdot 180^\circ}{n}.$$

# Example 4

Find the measure of each interior angle of a ceramic floor tile in the shape of an equiangular octagon (Figure 2.35).

Solution:

For an octagon, $n$ = 8.

Applying Corollary 2.5.3,

$$I = \frac{(8 - 2) \cdot 180}{8}$$

$$= \frac{6 \cdot 180}{8}$$

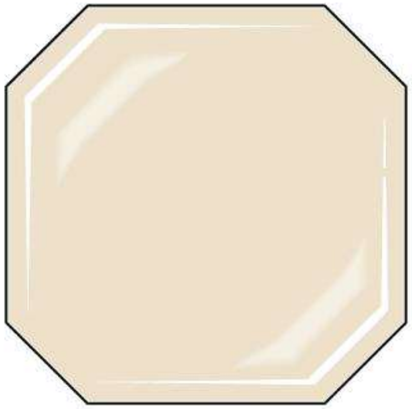


**Figure 2.35**

# Example 4 – *Solution*

cont'd

$$= \frac{1080}{8},$$

so $I = 135°$

Each interior angle of the tile measures 135°.

# Regular Polygons

Corollary 2.5.4

The sum of the measures of the four interior angles of a quadrilateral is 360°.

Corollary 2.5.5

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

Corollary 2.5.6

The measure *E* of each exterior angle of a regular polygon or equiangular polygon of *n* sides is $E = \frac{360^\circ}{n}$.

POLYGRAMS

# Polygrams

A **polygram** is the star-shaped figure that results when the sides of convex polygons with five or more sides are extended.

When the polygon is regular, the resulting polygram is also regular—that is, the interior acute angles are congruent, the interior reflex angles are congruent, and all sides are congruent.

The names of polygrams come from the names of the polygons whose sides were extended.

# Polygrams

Figure 2.37 shows a pentagram, a hexagram, and an octagram. With congruent angles and sides indicated, these figures are **regular polygrams**.

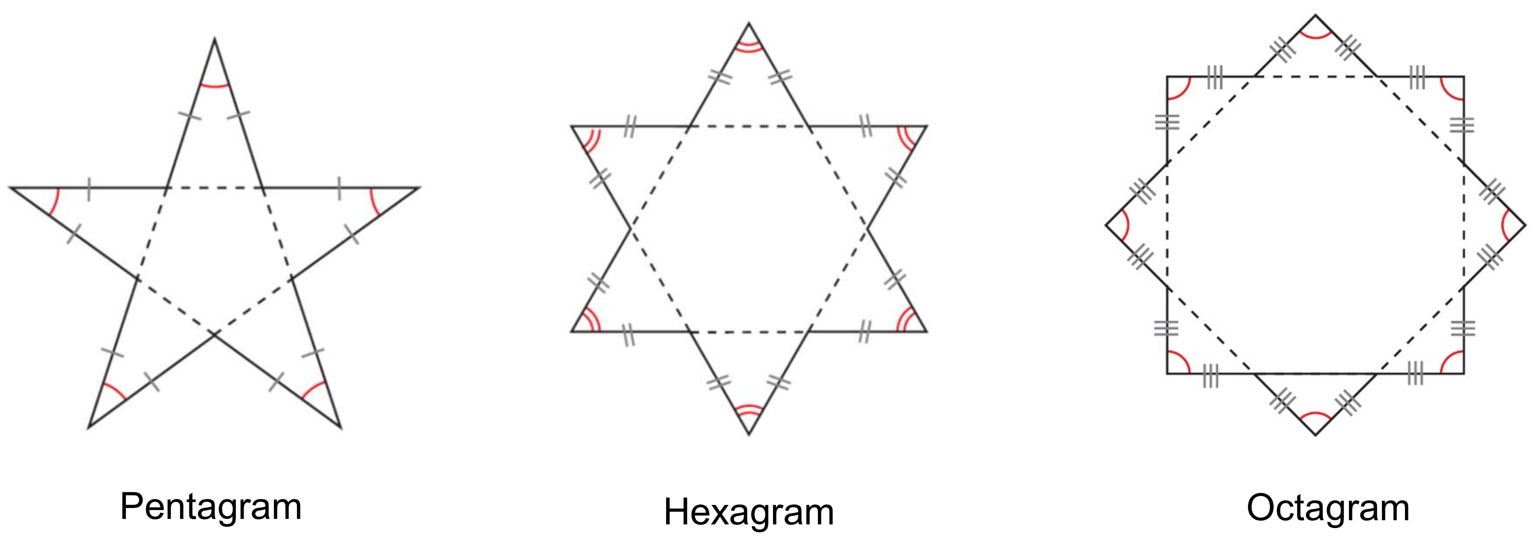



**Figure 2.37**