Quadratic Equations and Inequalities

CHAPTER 9

Digital Vision

9.6 Applications of Quadratic Functions

Objectives

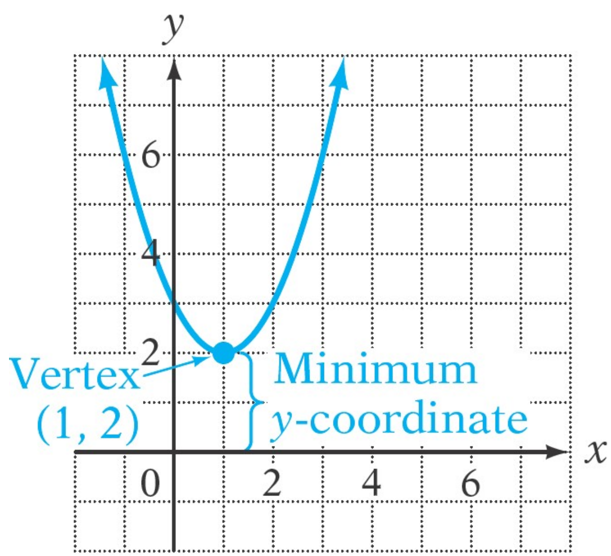
1 Minimum and maximum problems

2 Applications of minimum and maximum

Minimum and maximum problems

Minimum and maximum problems

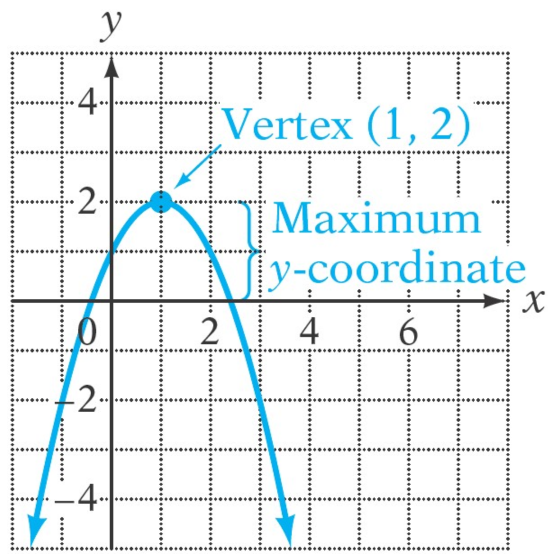
The graph of $f(x) = x^2 - 2x + 3$ is shown below. Because a is positive, the parabola opens up.



The vertex of the parabola is the lowest point on the parabola. It is the point that has the minimum *y*-coordinate. Therefore, the value of the function at this point is a **minimum.**

Minimum and maximum problems

The graph of $f(x) = –x^2 + 2x + 1$ is shown below. Because a is negative, the parabola opens down.



The vertex of the parabola is the highest point on the parabola. It is the point that has the maximum y-coordinate. Therefore, the value of the function at this point is a **maximum.**

Minimum and maximum problems

To find the minimum or maximum value of a quadratic function, first find the x-coordinate of the vertex.

Then evaluate the function at that value.

Example 1

Find the maximum or minimum value of the function $f(x) = -2x^2 + 4x + 3$.

Solution:

$$x = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$$

Find the *x*-coordinate of the vertex.
$a = -2, b = 4$

$$f(x) = -2x^2 + 4x + 3$$

$$f(1) = -2(1)^2 + 4(1) + 3$$

Evaluate the function at $x = 1$.

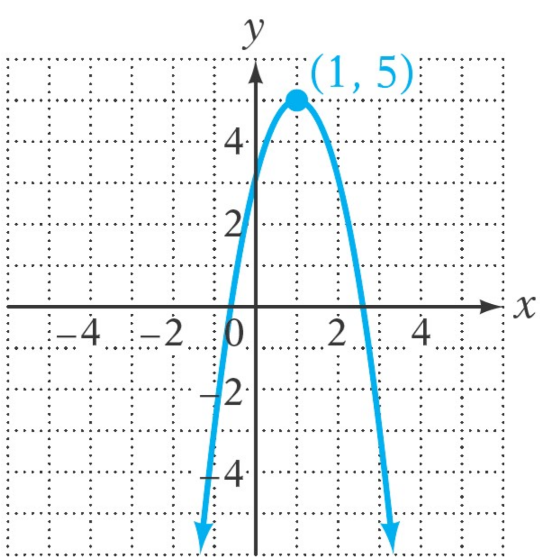
$$f(1) = 5$$

Example 1 – *Solution*

cont'd

Because $a < 0$, the graph of f opens down. Therefore, the function has a maximum value.

The maximum value of the function is 5. See the graph below.



$f(x) = -2x^2 + 4x + 3$

Applications of minimum and maximum

Example 2

A mining company has determined that the cost *c*, in dollars per ton, of mining a mineral is given by

$$c(x) = 0.2x^2 - 2x + 12$$

where *x* is the number of tons of the mineral that are mined. Find the number of tons of the mineral that should be mined to minimize the cost. What is the minimum cost?

Strategy:

- To find the number of tons that will minimize the cost, find the *x*-coordinate of the vertex.
- To find the minimum cost, evaluate the function at the *x*-coordinate of the vertex.

Example 2 – *Solution*

$$x = -\frac{b}{2a} = -\frac{-2}{2(0.2)} = 5$$

To minimize the cost, 5 tons should be mined.

$$c(x) = 0.2x^2 - 2x + 12$$

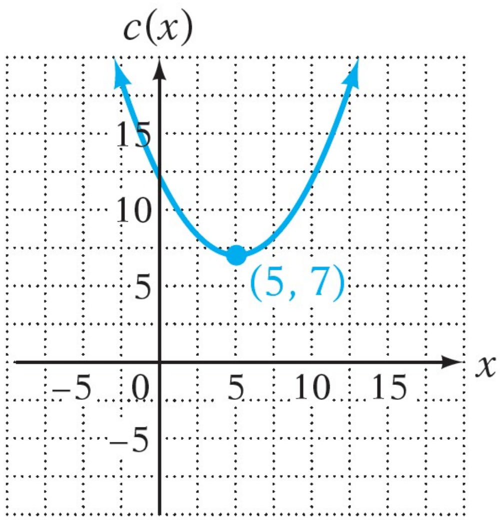
$$c(5) = 0.2(5)^2 - 2(5) + 12 = 5 - 10 + 12 = 7$$

The minimum cost per ton is $7.

Example 2 – *Solution*

cont'd

Note: The graph of the function $c(x) = 0.2x^2 - 2x + 12$ is shown below.



The vertex of the parabola is (5, 7). For any value of x less than 5, the cost per ton is greater than \$7. For any value of x greater than 5, the cost per ton is greater than \$7.7 is the minimum value of the function, and the minimum value occurs when $x = 5$.

Example 3

Find two numbers whose difference is 10 and whose product is a minimum.

Strategy:

- Let x and y represent the two numbers.

- Express y in terms of x.

$$y - x = 10 \qquad \text{The difference of the numbers is 10.}$$

$$y = x + 10 \qquad \text{Solve for } y.$$

Example 3

cont'd

- Express the product of the numbers in terms of x.

$$xy = x(x + 10) \qquad y = x + 10$$

$$f(x) = x^2 + 10x \qquad \text{The function } f \text{ represents the product of the two numbers.}$$

- To find one of the two numbers, find the x-coordinate of the vertex of $f(x) = x^2 + 10x$.

- To find the other number, replace x in $x + 10$ by the x-coordinate of the vertex and evaluate.

Example 3 – *Solution*

$$x = -\frac{b}{2a} = -\frac{10}{2(1)} = -5$$

$$x + 10 = -5 + 10 = 5$$

The numbers are –5 and 5.