Quadratic Equations and Inequalities

CHAPTER 9

9.1 Solving Quadratic Equations by Factoring or by Taking Square Roots

Objectives

1 Solve quadratic equations by factoring

2 Solve quadratic equations by taking square roots

Solve quadratic equations by factoring

Solve quadratic equations by factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where a and b are coefficients, c is a constant, and $a \neq 0$.

A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero.

Because the degree of the polynomial $ax^2 + bx + c = 0$ is 2, a quadratic equation is also called a **second-degree equation.**

The Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors equals zero.

Solve quadratic equations by factoring

PRINCIPLE OF ZERO PRODUCTS

If the product of two factors is zero, then at least one of the factors equals zero. If $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLES

1. Suppose $3x = 0$. The factors are 3 and x. The product equals zero, so at least one of the factors must be zero. Because $3 \neq 0$, we know that $x = 0$.
2. Suppose $-4(x - 4) = 0$. The factors are -4 and $x - 4$. The product equals zero, so at least one of the factors must be zero. Beause $-4 \neq 0$, we know that $x - 4 = 0$, which means $x = 4$.
3. Suppose $(x - 2)(x + 3) = 0$. The factors are $x - 2$ and $x + 3$. The product equals zero, so $x - 2 = 0$ or $x + 3 = 0$. If $x - 2 = 0$, then $x = 2$. If $x + 3 = 0$, then $x = -3$.

The Principle of Zero Products can be used to solve some quadratic equations.

Example 1

Solve by factoring: $2x^2 - 3x = 2$

Solution:

$$2x^2 - 3x = 2$$

$$2x^2 - 3x - 2 = 0$$

Write the equation in standard form.

$$(2x + 1)(x - 2) = 0$$

Factor the trinomial.

$$2x + 1 = 0 \qquad x - 2 = 0$$

$$2x = -1 \qquad x = 2$$

$$x = -\frac{1}{2}$$

Use the Principle of Zero Products. The product of $2x + 1$ and $x - 2$ is 0. Therefore, at least one of the factors is zero.

The solutions are $-\frac{1}{2}$ and 2.

Example 2

Solve by factoring: $(x+1)(2x-1)=2x+2$

Solution:

$$(x+1)(2x-1)=2x+2$$

$$2x^2+x-1=2x+2$$ Multiply the factors on the left side.

$$2x^2-x-3=0$$ Write the equation in standard form.

$$(x+1)(2x-3)=0$$ Factor.

$$x+1=0 \qquad 2x-3=0$$ Use the Principle of Zero Products.

$$x=-1 \qquad x=\frac{3}{2}$$

The solutions are –1 and $\frac{3}{2}$.

Solve quadratic equations by factoring

When a quadratic equation has two solutions that are the same number, the solution is called a **double root** of the equation.

The Principle of Zero Products also can be used to write an equation that has specific roots. For instance, suppose r and s are given as solutions of an equation. Then one possible equation is $(x - r)(x - s) = 0$, as shown below.

$$(x - r)(x - s) = 0$$

Use the Principle of Zero Products.

$$x - r = 0 \qquad\qquad x - s = 0$$

Solve quadratic equations by factoring

Solve for *x*.

$x = r$ $\qquad\qquad$ $x = s$

The solutions are *r* and *s*.

Given two solutions *r* and *s* and the equation $(x - r)(x - s) = 0$, we can find a quadratic equation that has the given solutions.

Example 3

Write a quadratic equation that has integer coefficients and has solutions $\frac{2}{3}$ and $\frac{1}{2}$.

Solution:

$$(x - r)(x - s) = 0$$

$$\left(x - \frac{2}{3}\right)\left(x - \frac{1}{2}\right) = 0$$

Replace r by $\frac{2}{3}$ and s by $\frac{1}{2}$.

$$x^2 - \frac{7}{6}x + \frac{1}{3} = 0$$

Multiply the binomials.

Example 3 – *Solution*

cont'd

$$6\left(x^2 - \frac{7}{6}x + \frac{1}{3}\right) = 6 \cdot 0$$

Multiply each side of the equation by 6, the LCD.

$$6x^2 - 7x + 2 = 0$$

A quadratic equation with solutions $\frac{2}{3}$ and $\frac{1}{2}$ is $6x^2 - 7x + 2 = 0$.

Solve quadratic equations by taking square roots

Solve quadratic equations by taking square roots

If *x* is a variable that can be positive or negative, then $\sqrt{x^2} = |x|$. This fact is used to solve a quadratic equation by taking square roots.

An equation containing the square of a binomial can be solved by taking square roots.

Example 4

Solve by taking square roots: $3(x-2)^2+12=0$

Solution:

$$3(x-2)^2+12=0$$

$$3(x-2)^2=-12$$ Solve for $(x-2)^2$.

$$(x-2)^2=-4$$

$$\sqrt{(x-2)^2}=\sqrt{-4}$$ Take the square root of each side of the equation. Then simplify.

$$|x-2|=2i$$

Example 4 – *Solution*

cont'd

$$x - 2 = \pm 2i$$

$$x - 2 = 2i \qquad x - 2 = -2i$$

Solve for x.

$$x = 2 + 2i \qquad x = 2 - 2i$$

The solutions are $2 + 2i$ and $2 - 2i$.