CHAPTER 8

Rational Exponents and Radicals

Digital Vision

8.4

Solving Equations Containing Radical Expressions

Objectives

1 Solve equations containing one or more radical expressions

2 Application problems

Solve equations containing one or more radical expressions

Solve equations containing one or more radical expressions

An equation that contains a variable expression in a radicand is a **radical equation**.

$$\left.\begin{aligned} \sqrt[3]{2x-5}+x&=7 \\ \sqrt{x+1}-\sqrt{x}&=4 \end{aligned}\right\}\text{Radical equations}$$

The following property is used to solve a radical equation.

PROPERTY OF RAISING EACH SIDE OF AN EQUATION TO A POWER

If two numbers are equal, then the same powers of the numbers are equal.

$$\text{If } a=b, \text{ then } a^n=b^n.$$

Example 1

Solve. **A.** $\sqrt{3x-2}-8=-3$ **B.** $\sqrt[3]{3x-1}=-4$

Solution:

A. $\sqrt{3x-2}-8=-3$

$\sqrt{3x-2}=5$ Rewrite the equation so that the radical is alone on one side of the equation.

$(\sqrt{3x-2})^2=5^2$ Square each side of the equation.

$3x-2=25$ Solve the resulting equation.

$3x=27$

$x=9$

Example 1 – *Solution*

cont'd

Check:

$$\sqrt{3x - 2} - 8 = -3 \qquad \text{Check the solution.}$$

$\sqrt{3 \cdot 9 - 2} - 8$	-3
$\sqrt{27 - 2} - 8$	-3
$\sqrt{25} - 8$	-3
$5 - 8$	-3

$$-3 = -3$$

The solution is 9.

B. $\sqrt[3]{3x - 1} = -4$

$$\left(\sqrt[3]{3x - 1}\right)^3 = (-4)^3 \qquad \text{Cube each side of the equation.}$$

Example 1 – *Solution*

cont'd

$$3x - 1 = -64$$

Solve the resulting equation.

$$3x = -63$$

$$x = -21$$

Check:

$$\sqrt[3]{3x - 1} = -4$$

Check the solution.

$\sqrt[3]{3(-21) - 1}$	-4
$\sqrt[3]{-63 - 1}$	-4
$\sqrt[3]{-64}$	-4

$$-4 = -4$$

The solution is –21.

Solve equations containing one or more radical expressions

When you raise both sides of an equation to an even power, the resulting equation may have a solution that is an extraneous solution of the original equation.

Therefore, it is necessary to check all proposed solutions of a radical equation.

Example 2

Solve. **A.** $x + 2\sqrt{x - 1} = 9$ **B.** $\sqrt{x + 7} = \sqrt{x} + 1$

Solution:

A. $x + 2\sqrt{x - 1} = 9$

$$2\sqrt{x - 1} = 9 - x$$

Rewrite the equation with the radical alone on one side of the equation.

$$(2\sqrt{x - 1})^2 = (9 - x)^2$$

Square each side of the equation.

$$4(x - 1) = 81 - 18x + x^2$$

$$4x - 4 = 81 - 18x + x^2$$

Example 2 – *Solution*

cont'd

$$0 = x^2 - 22x + 85$$

Write the quadratic equation in standard form.

$$0 = (x - 5)(x - 17)$$

Factor.

$$x - 5 = 0 \qquad x - 17 = 0$$

$$x = 5 \qquad\qquad x = 17$$

Use the Principle of Zero Products.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{x + 2\sqrt{x - 1} = 9} \\ \hline 5 + 2\sqrt{5 - 1} & 9 \\ 5 + 2\sqrt{4} & 9 \\ 5 + 2 \cdot 2 & 9 \\ 5 + 4 & 9 \\ \multicolumn{2}{r}{9 = 9} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{x + 2\sqrt{x - 1} = 9} \\ \hline 17 + 2\sqrt{17 - 1} & 9 \\ 17 + 2\sqrt{16} & 9 \\ 17 + 2 \cdot 4 & 9 \\ 17 + 8 & 9 \\ \multicolumn{2}{r}{25 \neq 9} \end{array}$$

Example 2 – *Solution*

cont'd

17 does not check as a solution. It is an extraneous solution of the equation.

The solution is 5.

B. $\sqrt{x + 7} = \sqrt{x} + 1$ A radical appears on each side of the equation.

$(\sqrt{x + 7})^2 = (\sqrt{x} + 1)^2$ Square each side of the equation.

$x + 7 = x + 2\sqrt{x} + 1$

$6 = 2\sqrt{x}$ Simplify the resulting equation.

$3 = \sqrt{x}$ The equation contains a radical.

Example 2 – *Solution*

cont'd

$$3^2 = (\sqrt{x})^2$$ Square each side of the equation.

$$9 = x$$

Check:

$$\sqrt{x+7} = \sqrt{x} + 1$$ Check the solution.

$\sqrt{9+7}$	$\sqrt{9}+1$
$\sqrt{16}$	$3+1$

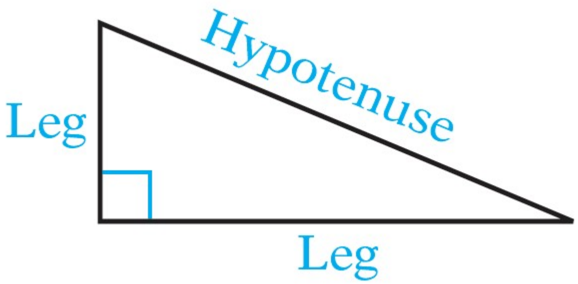
$$4 = 4$$

The solution is 9.

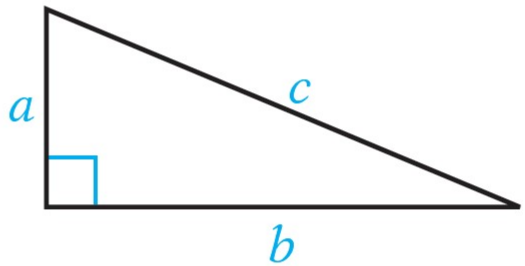
Application problems

Application problems

A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse**. The other two sides are called **legs**.



Pythagoras, a Greek mathematician, is credited with the discovery that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.



$$c^2 = a^2 + b^2$$

This is called the Pythagorean Theorem.

Application problems

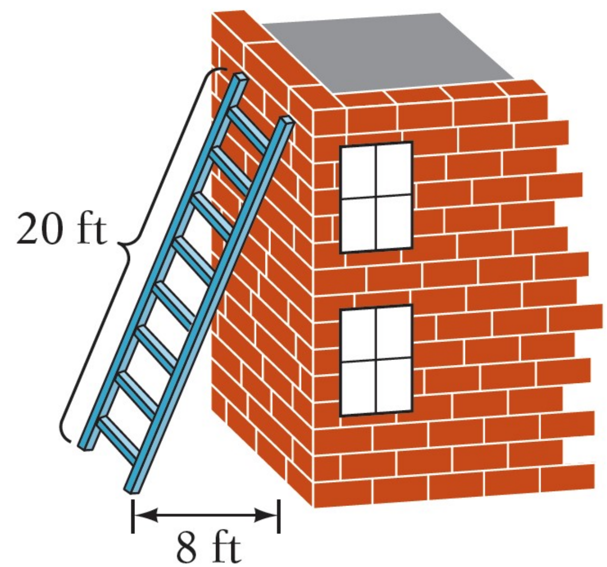
PYTHAGOREAN THEOREM

The square of the hypotenuse c of a right triangle is equal to the sum of the squares of the two legs, a and b.

$$c^2 = a^2 + b^2$$

Example 3

A ladder 20 ft long is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 8 ft from the building? Round to the nearest tenth.



Strategy:

To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Example 3 – *Solution*

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
20^2 &= 8^2 + b^2 \\
400 &= 64 + b^2 \\
336 &= b^2 \\
\sqrt{336} &= \sqrt{b^2} \\
\sqrt{336} &= b \\
18.3 &\approx b
\end{aligned}
$$

The distance is 18.3 ft.