CHAPTER

7

Rational Expressions

7.4 Rational Equations

Objectives

1. Solve fractional equations
2. Work problems
3. Uniform motion problems

Solve fractional equations

Solve fractional equations

To solve an equation containing fractions, **clear denominators** by multiplying each side of the equation by the LCD of the fractions. Then solve for the variable.

Occasionally, a value of the variable that appears to be a solution will make one of the denominators zero. Such a solution is called an **extraneous solution**.

In such a case, the equation has no solution for that value of the variable.

Solve fractional equations

Multiplying each side of an equation by a variable expression may produce an equation with different solutions from the original equation.

Thus, any time you multiply each side of an equation by a variable expression, you must check the resulting solution.

Example 1

Solve. **A.** $\dfrac{1}{4} = \dfrac{5}{x + 5}$ **B.** $\dfrac{2x}{x - 2} = \dfrac{1}{3x - 4} + 2$

Solution:

A. $$\frac{1}{4} = \frac{5}{x + 5}$$

$$4(x + 5)\frac{1}{4} = 4(x + 5)\left(\frac{5}{x + 5}\right)$$

Multiply each side of the equation by the LCD.

$$x + 5 = 4(5)$$

$$x + 5 = 20$$

Example 1 – *Solution*

cont'd

$$x = 15$$

15 checks as a solution.

The solution is 15.

B. $$\frac{2x}{x-2} = \frac{1}{3x-4} + 2$$

$$(x-2)(3x-4)\left(\frac{2x}{x-2}\right) = (x-2)(3x-4)\left(\frac{1}{3x-4} + 2\right)$$

$$(3x-4)2x = (x-2)(3x-4)\left(\frac{1}{3x-4}\right) + (x-2)(3x-4)2$$

Example 1 – *Solution*

cont'd

$$6x^2 - 8x = x - 2 + 6x^2 - 20x + 16$$

$$6x^2 - 8x = 6x^2 - 19x + 14$$

$$11x = 14$$

$$x = \frac{14}{11}$$

$\frac{14}{11}$ checks as a solution.

The solution is $\frac{14}{11}$.

Work problems

Work problems

Rate of work is that part of a task that is completed in one unit of time. If a mason can build a retaining wall in 12 h, then in 1 h the mason can build $\frac{1}{12}$ of the wall.

The mason's rate of work is $\frac{1}{12}$ of the wall each hour. If an apprentice can build the wall in *x* hours, the rate of work for the apprentice is $\frac{1}{x}$ of the wall each hour. In solving a work problem, the goal is to determine the time it takes to complete a task.

The basic equation that is used to solve work problems is

$$\textbf{Rate of work} \times \textbf{Time worked} = \textbf{Part of task completed}$$

Work problems

For example, if a pipe can fill a tank in 5 h, then in 2 h the pipe will fill $\frac{1}{5} \times 2 = \frac{2}{5}$ of the tank.

In *t* hours, the pipe will fill $\frac{1}{5} \times t = \frac{t}{5}$ of the tank.

Work problems

STRATEGY FOR SOLVING A WORK PROBLEM

- For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

- Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1, the complete task.

Example 2

An electrician requires 12 h to wire a house. The electrician's apprentice can wire a house in 16 h. After working alone on a job for 4 h, the electrician quits, and the apprentice completes the task. How long does it take the apprentice to finish wiring the house?

Strategy:

- Time required for the apprentice to finish wiring the house: t

	Rate	$\cdot$	Time	$=$	Part
Electrician	$\frac{1}{12}$	$\cdot$	4	$=$	$\frac{4}{12}$
Apprentice	$\frac{1}{16}$	$\cdot$	t	$=$	$\frac{t}{16}$

Example 2

cont'd

- The sum of the part of the task completed by the electrician and the part of the task completed by the apprentice is 1.

Solution:

$$\frac{4}{12} + \frac{t}{16} = 1$$

$$\frac{1}{3} + \frac{t}{16} = 1$$

$$48\left(\frac{1}{3} + \frac{t}{16}\right) = 48(1)$$

Example 2 – *Solution*

cont'd

$$16 + 3t = 48$$

$$3t = 32$$

$$t = \frac{32}{3}$$

$$= 10\frac{2}{3}$$

It takes the apprentice $10\frac{2}{3}$ h to finish wiring the house.

Uniform motion problems

Uniform motion problems

A car that travels constantly in a straight line at 55 mph is in uniform motion. **Uniform motion** means that the speed of an object does not change.

The basic equation used to solve uniform motion problems is

$$\textbf{Distance} = \textbf{Rate} \times \textbf{Time}$$

An alternative form of this equation can be written by solving the equation for time. This form of the equation is used to solve the following problem.

$$\frac{\textbf{Distance}}{\textbf{Rate}} = \textbf{Time}$$

Uniform motion problems

STRATEGY FOR SOLVING A UNIFORM MOTION PROBLEM

- ► For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

- ► Determine how the times traveled by each object are related. For example, it may be known that the times are equal, or the total time may be known.

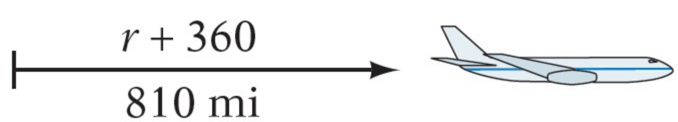
Example 3

A marketing executive traveled 810 mi on a corporate jet in the same amount of time that it took to travel an additional 162 mi by helicopter. The rate of the jet was 360 mph greater than the rate of the helicopter. Find the rate of the jet.

Strategy:

- Rate of the helicopter: r

Rate of the jet: $r + 360$



Example 3

cont'd

	Distance	$\div$	Rate	$=$	Time
Jet	810	$\div$	$r + 360$	$=$	$\dfrac{810}{r + 360}$
Helicopter	162	$\div$	r	$=$	$\dfrac{162}{r}$

- The time traveled by jet is equal to the time traveled by helicopter.

Example 3 – *Solution*

$$\frac{810}{r + 360} = \frac{162}{r}$$

$$r(r + 360)\left(\frac{810}{r + 360}\right) = r(r + 360)\left(\frac{162}{r}\right)$$

$$810r = (r + 360)162$$

$$810r = 162r + 58{,}320$$

$$648r = 58{,}320$$

$$r = 90$$

The rate of the helicopter was 90 mph.

Example 3 – *Solution*

cont'd

$$r + 360 = 90 + 360$$

$$= 450$$

Substitute the value of *r* into the variable expression for the rate of the jet.

The rate of the jet was 450 mph.