CHAPTER 7

Rational Expressions

7.1 Introduction to Rational Functions

Objectives

1 Find the domain of a rational function

2 Simplify rational expressions

Find the domain of a rational function

Find the domain of a rational function

An expression in which the numerator and denominator are polynomials is called a **rational expression**. Examples of rational expressions are shown below.

$$\frac{9}{z} \qquad \frac{3x+4}{2x^2+1} \qquad \frac{x^3-x+1}{x^2-3x-5}$$

The expression $\frac{\sqrt{x}+3}{x}$ is not a rational expression because $\sqrt{x}+3$ is not a polynomial.

A function that is written in terms of a rational expression is a **rational function**.

Find the domain of a rational function

Each of the following equations represents a rational function.

$$f(x) = \frac{x^2 + 3}{2x - 1} \qquad g(t) = \frac{3}{t^2 - 4} \qquad R(z) = \frac{z^2 + 3z - 1}{z^2 + z - 12}$$

To evaluate a rational function, replace the variable by its value. Then simplify.

Example 1

Given $f(x) = \dfrac{3x - 4}{x^2 - 2x + 1}$, find $f(-3)$.

Solution:

$$f(x) = \frac{3x - 4}{x^2 - 2x + 1}$$

$$f(-3) = \frac{3(-3) - 4}{(-3)^2 - 2(-3) + 1}$$ **Substitute −3 for *x*.**

$$f(-3) = \frac{-9 - 4}{9 + 6 + 1}$$

$$f(-3) = \frac{-13}{16}$$

$$f(-3) = -\frac{13}{16}$$

Find the domain of a rational function

DOMAIN OF A RATIONAL FUNCTION

Because division by zero is not defined, the **domain of a rational function** must exclude those numbers for which the value of the denominator is zero.

EXAMPLES

1. $f(x) = \dfrac{x + 3}{x - 4}$

The denominator equals 0 when $x - 4 = 0$, or $x = 4$. The domain of f is all real numbers except 4. This is written $\{x \mid x \neq 4\}$.

2. $f(x) = \dfrac{x^2 - 6}{4x + 8}$

The denominator equals 0 when $4x + 8 = 0$. Solve the equation for x.

$$4x + 8 = 0$$
$$4x = -8$$
$$x = -2$$

The domain of f is all real numbers except -2. This is written $\{x \mid x \neq -2\}$.

3. $f(x) = \dfrac{2x}{x^2 + 4}$

The dominator equals 0 when $x^2 + 4 = 0$. However, $x^2 \geq 0$ for all real numbers. Therefore, $x^2 + 4 > 0$ for all real numbers, and the denominator is never 0. The domain is all real numbers, or $\{x \mid x \in \text{ real numbers}\}$.

Example 2

Find the domain of $f(x) = \dfrac{2x - 6}{x^2 - 3x - 4}$.

Solution:

The domain must exclude values of *x* for which $x^2 - 3x - 4 = 0$. Solve this equation for *x*.

$$x^2 - 3x - 4 = 0$$

$$(x + 1)(x - 4) = 0$$ Factor the trinomial.

$$x + 1 = 0 \qquad x - 4 = 0$$ Use the Principle of Zero Products to set each factor equal to zero.

$$x = -1 \qquad x = 4$$

Example 2 – *Solution*

cont'd

The solutions are –1 and 4. The domain of *f* must exclude these values.

The domain of *f* is $\{x | x \neq -1, x \neq 4\}$.

Simplify rational expressions

Simplify rational expressions

A rational expression is in **simplest form** when the numerator and denominator have no common factors other than 1.

Example 3

Simplify.

A. $\dfrac{x^2 - 16}{x^2 + 11x + 28}$ **B.** $\dfrac{12 + 5x - 2x^2}{2x^2 - 3x - 20}$

Solution:

A. $$\frac{x^2 - 16}{x^2 + 11x + 28} = \frac{(x + 4)(x - 4)}{(x + 4)(x + 7)}$$

Factor the numerator and the denominator.

$$= \frac{\overset{1}{\cancel{(x + 4)}}(x - 4)}{\underset{1}{\cancel{(x + 4)}}(x + 7)}$$

Divide by the common factors.

$$= \frac{x - 4}{x + 7}$$

Write the answer in simplest form.

Example 3 – *Solution*

cont'd

B. $$\frac{12 + 5x - 2x^2}{2x^2 - 3x - 20} = \frac{(4 - x)(3 + 2x)}{(x - 4)(2x + 5)}$$

Factor the numerator and the denominator.

$$= \frac{\overset{-1}{\cancel{(4 - x)}}(3 + 2x)}{\underset{1}{\cancel{(x - 4)}}(2x + 5)}$$

Divide by the common factors. Remember that $4 - x = -(x - 4)$. Therefore, $\frac{4 - x}{x - 4} = \frac{-(x - 4)}{x - 4} = \frac{-1}{1} = -1$.

$$= -\frac{2x + 3}{2x + 5}$$

Write the answer in simplest form.