Conic Sections

CHAPTER 12

12.5 Quadratic Inequalities and Systems of Inequalities

Objectives

1. Graph the solution set of a quadratic inequality in two variables

2. Graph the solution set of a nonlinear system of inequalities

Graph the solution set of a quadratic inequality in two variables

Graph the solution set of a quadratic inequality in two variables

The **graph of a quadratic inequality in two** variables is a region of the plane that is bounded by one of the conic sections (parabola, circle, ellipse, or hyperbola).

When graphing an inequality of this type, first replace the inequality symbol with an equals sign.

Graph the resulting conic using a dashed curve when the original inequality is less than (<) or greater than (>).

Graph the solution set of a quadratic inequality in two variables

Use a solid curve when the original inequality is ≤ or ≥. Use the point with coordinates (0, 0) to determine which region of the plane to shade.

If (0, 0) is a solution of the inequality, then shade the region of the plane containing the point with coordinates (0, 0). If not, shade the other portion of the plane.

Example 1

Graph the solution set.

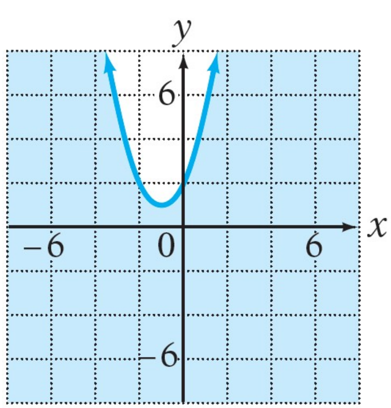
A. $y \leq x^2 + 2x + 2$ **B.** $\dfrac{y^2}{9} - \dfrac{x^2}{4} \geq 1$

Solution:

A. $y \leq x^2 + 2x + 2$

$y = x^2 + 2x + 2$

Change the inequality to an equality. This is the equation of a parabola that opens up. The coordinates of the vertex are $(-1, 1)$. The equation of the axis of symmetry is $x = -1$.



Because the inequality is $\leq$, the graph is drawn as a solid curve.

Substitute $(0, 0)$ into the inequality. Because the inequality $0 < 0^2 + 2(0) + 2$ is true, the point with coordinates $(0, 0)$ should be in the shaded region.

Example 1 – *Solution*

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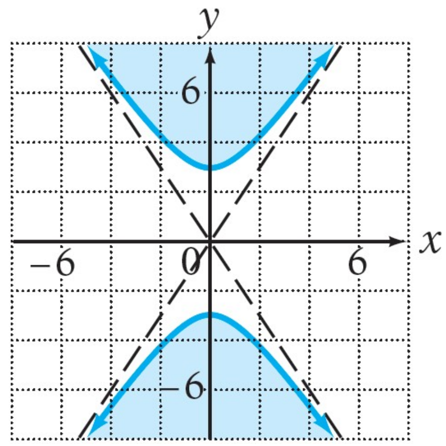
B. $\dfrac{y^2}{9} - \dfrac{x^2}{4} \geq 1$

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

Change the inequality to an equality. This is the equation of a hyperbola. The coordinates of the vertices are $(0, -3)$ and $(0, 3)$. The equations of the asymptotes are $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$.



Because the inequality is $\geq$, the graph is drawn as a solid curve.

Substitute (0, 0) into the inequality.

Because the inequality $\frac{0^2}{9} - \frac{0^2}{4} \geq 1$ is not true, the point with coordinates (0, 0) should not be in the shaded region.

Graph the solution set of a nonlinear system of inequalities

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A **nonlinear system of inequalities** is a system in which one or more of the inequalities is a nonlinear inequality. The **solution set of a nonlinear system of inequalities** is the intersection of the solution sets of the individual inequalities.

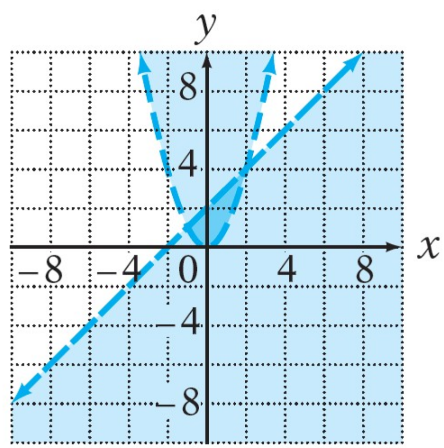
To graph the solution set of a system of inequalities, first graph the solution set for each inequality. The graph of the solution set of the system of inequalities is the region of the plane represented by the intersection of the two shaded regions.

Example 2

Graph the solution set. **A.** $y > x^2$ $\quad$ $y < x + 2$ $\qquad$ **B.** $\frac{x^2}{9} - \frac{y^2}{16} \geq 1$ $\quad$ $x^2 + y^2 \leq 4$

Solution:

A.



Graph the solution set of each inequality.

$y = x^2$ is the equation of a parabola. Use a dashed curve. Shade inside the parabola.

$y = x + 2$ is the equation of a line. Use a dashed line. Shade below the line.

The solution set is the region of the plane represented by the intersection of the solution sets of the individual inequalities.

Example 2 – *Solution*

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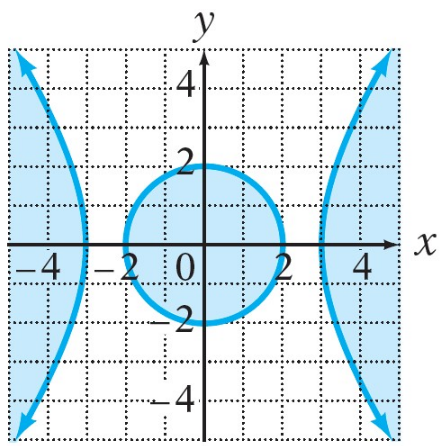
B.



Graph the solution set of each inequality.

$\frac{x^2}{9} - \frac{y^2}{16} = 1$ is the equation of a hyperbola. Use a solid curve. The point with coordinates (0, 0) should not be in the shaded region.

$x^2 + y^2 = 4$ is the equation of a circle. Use a solid curve. Shade inside the circle.

The solution sets of the two inequalities do not intersect.

The system of inequalities has no real number solutions.