Conic Sections

CHAPTER 12

12.2 The Circle

Objectives

1. Find the equation of a circle and then graph the circle
2. Write the equation of a circle in standard form and then graph the circle

Find the equation of a circle and then graph the circle

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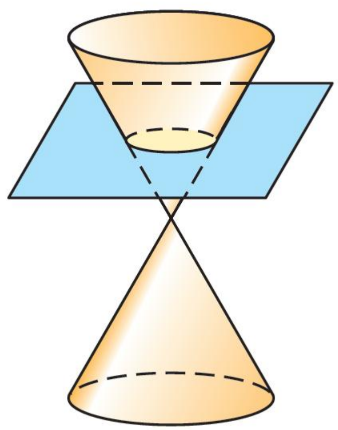
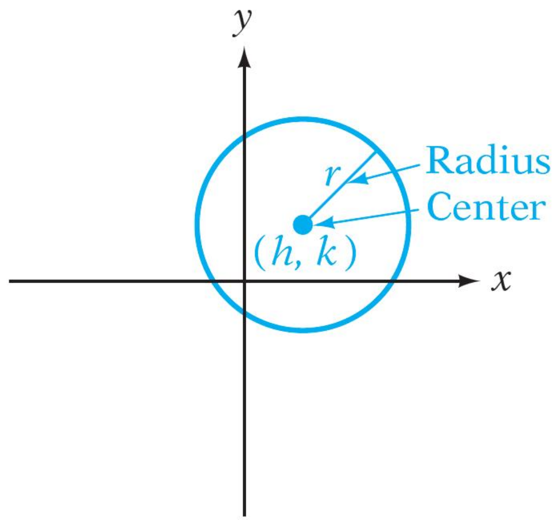
A *circle* is a conic section formed by the intersection of a cone and a plane that is parallel to the base of the cone.





A **circle** can be defined as all the points $P(x, y)$ in the plane that are a fixed distance from a given point $C(h, k)$ called the **center**. The fixed distance is the **radius** of the circle.

Find the equation of a circle and then graph the circle

The equation of a circle can be determined by using the distance formula.

STANDARD FORM OF THE EQUATION OF A CIRCLE

The standard form of the equation of a circle with center $C(h, k)$ and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLES

1. The equation $(x - 3)^2 + (y - 1)^2 = 6^2$ is the equation of a circle in standard form, with $h = 3$ and $k = 1$. Therefore, the coordinates of the center are (3, 1). Because $r = 6$, the radius of the circle is 6.
2. The equation $(x - 2)^2 + (y + 3)^2 = 16$ is not in standard form. In standard form, the equation is written as $(x - 2)^2 + [y - (-3)]^2 = 4^2$, with $h = 2$ and $k = -3$. Therefore, the coordinates of the center are $(2, -3)$. Because $r = 4$, the radius of the circle is 4.

Example 1

Find the equation of the circle that passes through the point *P*(–1, 4) and whose center is the point *C*(2, –3).

Solution:

The radius of the circle is the distance from the center *C* to the point *P*. Use the distance formula to find this distance.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{[2 - (-1)]^2 + (-3 - 4)^2} \qquad (x_1, y_1) = (-1, 4), \; (x_2, y_2) = (2, -3)$$

$$r = \sqrt{3^2 + (-7)^2}$$

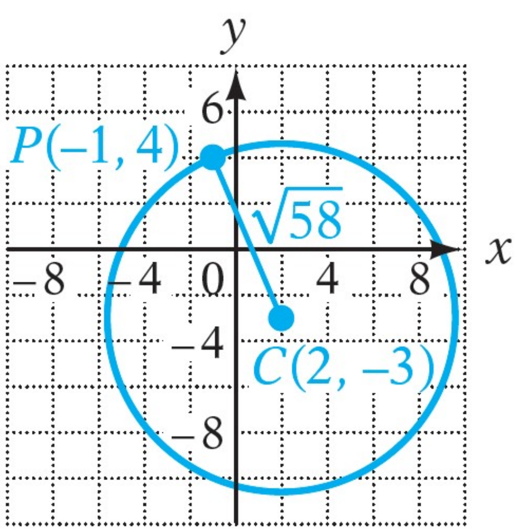
$$= \sqrt{9 + 49}$$

$$r = \sqrt{58}$$

$$(x - 2)^2 + [y - (-3)]^2 = (\sqrt{58})^2$$

$$(x - 2)^2 + (y + 3)^2 = 58$$

The radius of the circle is $\sqrt{58}$. Use the coordinates of the center $C(2, -3)$ and the radius to write the equation.



Write the equation of a circle in standard form and then graph the circle

Write the equation of a circle in standard form and then graph the circle

The equation of a circle can also be expressed in **general form** as

$$x^2 + y^2 + ax + by + c = 0$$

To rewrite this equation in standard form, it is necessary to complete the square on the *x* and *y* terms.

Example 3

Write the equation of the circle $x^2 + y^2 + 3x - 2y = 1$ in standard form. Then sketch its graph.

Solution:

$$x^2 + y^2 + 3x - 2y = 1$$

$$(x^2 + 3x) + (y^2 - 2y) = 1$$

Group terms involving x and terms involving y.

$$\left(x^2 + 3x + \frac{9}{4}\right) + (y^2 - 2y + 1) = 1 + \frac{9}{4} + 1$$

Complete the square on $x^2 + 3x$ and $y^2 - 2y$.

$$\left(x + \frac{3}{2}\right)^2 + (y - 1)^2 = \frac{17}{4}$$

Factor each trinomial.

Example 3 – *Solution*

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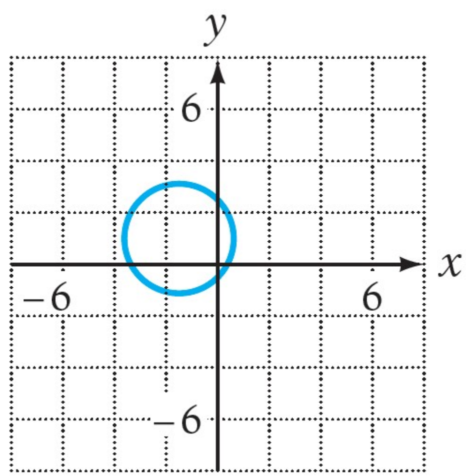

Draw a circle with center $\left(-\frac{3}{2}, 1\right)$ and radius $\sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2} \approx 2.1.$