CHAPTER

11

Exponential and Logarithmic Functions

Digital Vision

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11.4 Exponential and Logarithmic Equations

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Objectives

1 Solve exponential equations

2 Solve logarithmic equations

Solve exponential equations

Solve exponential equations

An **exponential equation** is one in which a variable occurs in an exponent.

The examples below are exponential equations.

$$6^{2x+1} = 6^{3x-2}$$
$$4^{x} = 3$$
$$2^{x+1} = 7$$

An exponential equation in which each side of the equation can be expressed in terms of the same base can be solved by using the Equality of Exponents Property.

Solve exponential equations

The Equality of Exponents Property states that

If $b^u = b^v$, then $u = v$.

If the bases are not the same, as in next Example, try to rewrite the equation so that both sides are written in terms of the same base.

Example 1

Solve and check: $9^{x+1} = 27^{x-1}$

Solution:

$$9^{x+1} = 27^{x-1}$$

$$(3^2)^{x+1} = (3^3)^{x-1}$$

Rewrite each side of the equation using the same base.

$$3^{2x+2} = 3^{3x-3}$$

$$2x + 2 = 3x - 3$$

Use the Equality of Exponents Property to equate the exponents.

$$2 = x - 3$$

Solve the resulting equation.

$$5 = x$$

Example 1 – *Solution*

cont'd

Check:

9^{x+1}	$= 27^{x-1}$
9^{5+1}	27^{5-1}
9^{6}	27^{4}
$(3^{2})^{6}$	$(3^{3})^{4}$

$$3^{12} = 3^{12}$$

The solution is 5.

Solve exponential equations

When both sides of an exponential equation cannot easily be expressed in terms of the same base, logarithms are used to solve the exponential equation.

Example 2

Solve for x. Round to the nearest ten-thousandth.

A. $4^x = 7$ **B.** $3^{2x} = 4$

Solution:

A.

$$4^x = 7$$

$$\log 4^x = \log 7$$ Take the common logarithm of each side of the equation.

$$x \log 4 = \log 7$$ Rewrite using the properties of logarithms.

$$x = \frac{\log 7}{\log 4}$$ Solve for x.

$$x \approx 1.4037$$

The solution is 1.4037.

Example 2 – *Solution*

cont'd

B. $$3^{2x} = 4$$

$$\log 3^{2x} = \log 4$$ Take the common logarithm of each side of the equation.

$$2x \log 3 = \log 4$$ Rewrite using the properties of logarithms.

$$x = \frac{\log 4}{2 \log 3}$$ Solve for *x*.

$$x \approx 0.6309$$

The solution is 0.6309.

Solve logarithmic equations

Solve logarithmic equations

A logarithmic equation can be solved by using the properties of logarithms.

Example 4

Solve: $\log_4(x^2 - 6x) = 2$

Solution:

$$\log_4(x^2 - 6x) = 2$$

$$4^2 = x^2 - 6x$$

Rewrite the equation in exponential form.

$$16 = x^2 - 6x$$

Simplify.

$$0 = x^2 - 6x - 16$$

Write the quadratic equation in standard form.

$$0 = (x + 2)(x - 8)$$

Factor and use the Principle of Zero Products.

Example 4 – *Solution*

cont'd

$$x + 2 = 0 \qquad x - 8 = 0$$

$$x = -2 \qquad x = 8$$

–2 and 8 check as solutions. The solutions are –2 and 8.

Solve logarithmic equations

Some logarithmic equations can be solved by using the 1–1 Property of Logarithms, which states that for $x > 0$, $y > 0$, $b > 0$, $b \neq 1$,

$$\text{If } \log_b x = \log_b y, \text{ then } x = y.$$

Example 5

Solve: $\log_2 x - \log_2(x - 1) = \log_2 2$

Solution:

$$\log_2 x - \log_2(x - 1) = \log_2 2$$

$$\log_2\left(\frac{x}{x - 1}\right) = \log_2 2$$

Use the Quotient Property of Logarithms.

$$\frac{x}{x - 1} = 2$$

Use the 1–1 Property of Logarithms.

$$(x - 1)\left(\frac{x}{x - 1}\right) = (x - 1)2$$

Solve for *x*.

Example 5 – *Solution*

cont'd

$$x = 2x - 2$$

$$-x = -2$$

$$x = 2$$

2 checks as a solution. The solution is 2.

Solve logarithmic equations

Some logarithmic equations cannot be solved algebraically. In such cases, a graphical approach may be appropriate.