CHAPTER 11

Exponential and Logarithmic Functions

11.1 Exponential Functions

Objectives

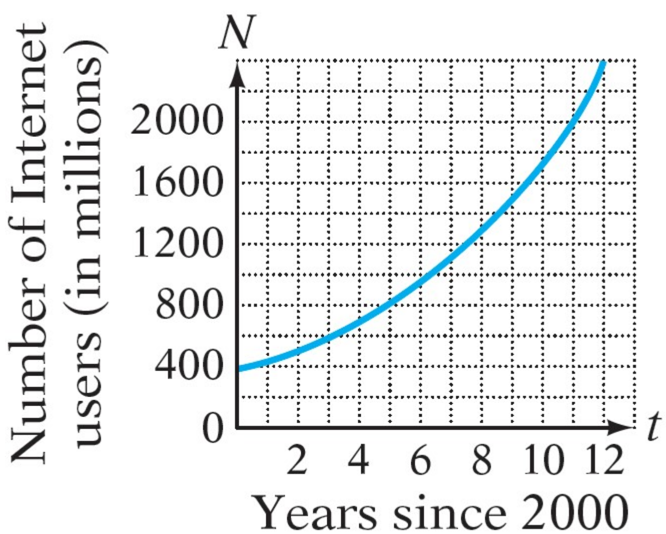
1 Evaluate exponential functions

2 Graph exponential functions

Evaluate exponential functions

Evaluate exponential functions

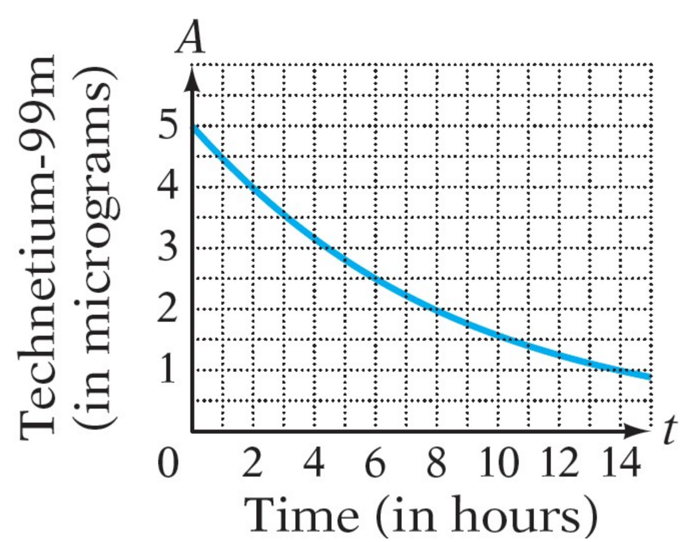
Data suggest that since the year 2000, the number of Internet users worldwide has been increasing at a rate of approximately 17% per year. The graph at the right shows this growth. This graph depicts an example of **exponential growth**.



Evaluate exponential functions

Nuclear medicine physicians use radioisotopes for the diagnosis and treatment of certain diseases. One of the most widely used isotopes is technetium-99m. One use of this isotope is in the diagnosis of cardiovascular disease.

The graph at the right shows the amount of technetium-99m in a patient after its injection into the patient. This graph depicts an example of **exponential decay**.



Evaluate exponential functions

DEFINITION OF AN EXPONENTIAL FUNCTION

The **exponential function** with **base *b*** is defined by

$$f(x) = b^x$$

where $b > 0$, $b \neq 1$, and x is any real number.

In the definition of an exponential function, *b*, the base, is required to be positive. If the base were a negative number, the value of the function would be a complex number for some values of *x*.

Evaluate exponential functions

For instance, the value of $f(x) = (-4)^x$ when $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = (-4)^{\frac{1}{2}} = \sqrt{-4} = 2i$. To avoid complex number values of a function, the base of the exponential function is a positive number.

Because $f(x) = b^x \ (b > 0, b \neq 1)$ can be evaluated at both rational and irrational numbers, the domain of *f* is all real numbers. And because $b^x > 0$ for all values of *x*, the range of *f* is the positive real numbers.

Example 1

Evaluate $f(x) = \left(\frac{1}{2}\right)^x$ at $x = 2$ and $x = -3$.

Solution:

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$f(2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$f(-3) = \left(\frac{1}{2}\right)^{-3} = 2^3 = 8$$

Evaluate exponential functions

A frequently used base in applications of exponential functions is an irrational number designated by *e*. The number *e* is approximately 2.71828183. It is an irrational number, so it has a nonterminating, nonrepeating decimal representation.

NATURAL EXPONENTIAL FUNCTION

The function defined by $f(x) = e^x$ is called the **natural exponential function.**

The $\boxed{e^x}$ key on a calculator can be used to evaluate the natural exponential function.

Graph exponential functions

Graph exponential functions

Some of the properties of an exponential function can be seen by considering its graph.

EXPONENTIAL FUNCTIONS ARE 1–1

The exponential function defined by $f(x) = b^x$, $b > 0$, $b \neq 1$, is a 1–1 function.

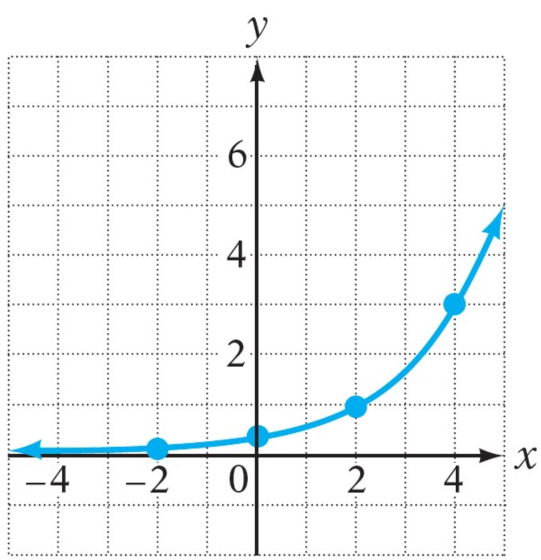
Example 4

Graph **A.** $f(x) = 3^{\frac{1}{2}x-1}$ **B.** $f(x) = 2^x - 1$

Solution:

A.

x	$y = 3^{\frac{1}{2}x-1}$
-2	$\frac{1}{9}$
0	$\frac{1}{3}$
2	1
4	3



Example 4 – *Solution*

cont'd

B.

x	$y = 2^x - 1$
-2	$-\frac{3}{4}$
-1	$-\frac{1}{2}$
0	0
1	1
2	3
3	7