

Functions and Relations

CHAPTER

10

Digital Vision

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10.1

Translations of Graphs

Objective

- 1 Graph by using translations



Graph by using translations

Graph by using translations

The graphs of $f(x) = |x|$ and $g(x) = |x| + 2$ are shown in Figure 1.

Note that for a given x -coordinate, the y -coordinate on the graph of g is 2 units higher than that on the graph of f .

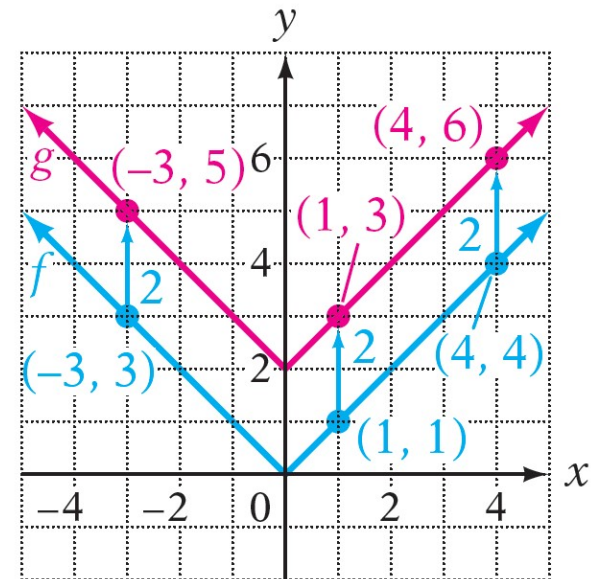


Figure 1

The graph of g is said to be a **vertical translation**, or **vertical shift**, of the graph of f .

Graph by using translations

VERTICAL TRANSLATION

If f is a function and c is a positive constant, then

the graph of $y = f(x) + c$ is the graph of $y = f(x)$ shifted up c units.

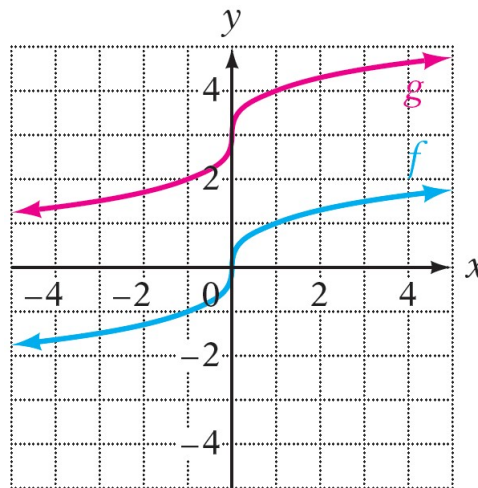
the graph of $y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.

Example 1

Given the graph of the function $y = f(x)$ shown below in blue, graph $g(x) = f(x) + 3$ by using a vertical translation.

Solution:

The graph of $g(x) = f(x) + 3$ is the graph of $y = f(x)$ shifted 3 units up. This graph is shown below in red.



Graph by using translations

The graphs of $f(x) = |x|$ and $g(x) = |x - 2|$ are shown in Figure 3.

Note that the graph of g is the graph of f shifted 2 units to the right. The graph of g is a **horizontal translation**, or **horizontal shift**, of the graph of f .

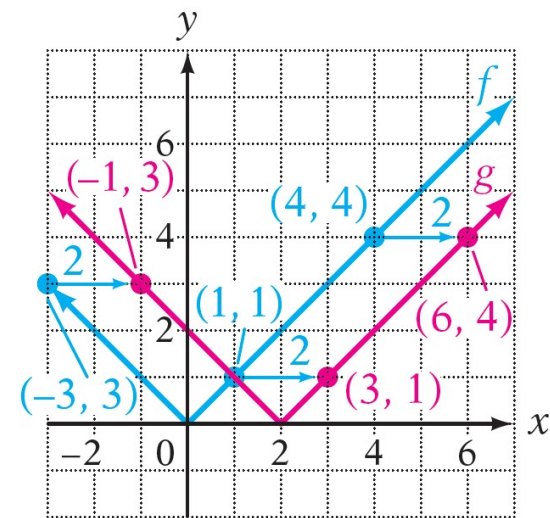


Figure 3

In this situation, each x -coordinate is moved to the right 2 units, but the y -coordinates are unchanged.

Graph by using translations

HORIZONTAL TRANSLATION

If f is a function and c is a positive constant, then

the graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.

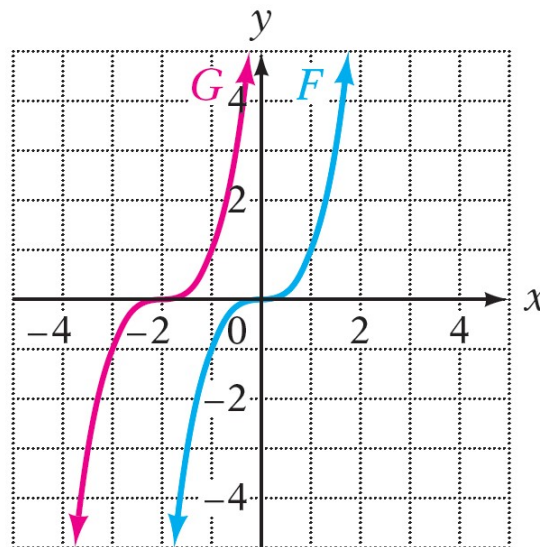
the graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.

Example 2

Given the graph of the function $y = F(x)$ shown at the right in blue, graph $G(x) = F(x + 2)$ by using a horizontal translation.

Solution:

The graph of G (shown in red) is the graph of F shifted horizontally 2 units to the left.





Graph by using translations

It is possible for a graph to involve both a horizontal and a vertical translation.

Example 3

Given the graph of the function $y = f(x)$ shown below in blue, graph $A(x) = f(x + 1) - 3$ by using both a horizontal and a vertical translation.

Solution:

The graph of A includes a horizontal shift of 1 unit to the left and a vertical shift of 3 units down. The graph of A is shown in red at the right.

