CHAPTER 7

Rational Expressions

7.5 Proportions and Variation

Objectives

1 Proportions

2 Variation problems

Proportions

Proportions

Quantities such as 3 feet, 5 liters, and 2 miles are number quantities written with units. In these examples, the units are feet, liters, and miles.

A **ratio** is the quotient of two quantities that have the same unit.

The weekly wages of a painter are \$800. The painter spends \$150 a week for food. The ratio of wages spent for food to total weekly wages is written as shown below.

$$\frac{\$150}{\$800} = \frac{150}{800} = \frac{3}{16}$$

A ratio is in simplest form when the two numbers do not have a common factor. The units are not written.

Proportions

A **rate** is the quotient of two quantities that have different units.

A car travels 120 mi on 3 gal of gas. The miles-to-gallon rate is written as shown below.

$$\frac{120 \text{ mi}}{3 \text{ gal}} = \frac{40 \text{ mi}}{1 \text{ gal}}$$

A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

Proportions

A **proportion** is an equation that states that two ratios or rates are equal.

For example, $\frac{90 \text{ km}}{4 \text{ L}} = \frac{45 \text{ km}}{2 \text{ L}}$ and $\frac{3}{4} = \frac{x + 2}{16}$ are proportions.

Note that a proportion is a special kind of fractional equation. Many application problems can be solved by using proportions.

Example 1

A stock investment of 50 shares pays a dividend of $106. At this rate, how many additional shares are needed to earn a dividend of $424?

Strategy:

To find the additional number of shares that are required, write and solve a proportion using x to represent the additional number of shares.

Then $50 + x$ is the total number of shares of stock.

Example 1 – *Solution*

$$\frac{106}{50} = \frac{424}{50 + x}$$

$$\frac{53}{25} = \frac{424}{50 + x}$$

Simplify the left side.

$$25(50 + x)\frac{53}{25} = 25(50 + x)\frac{424}{50 + x}$$

Multiply each side by the denominators.

$$(50 + x)53 = (25)424$$

$$2650 + 53x = 10{,}600$$

$$53x = 7950$$

$$x = 150$$

An additional 150 shares of stock are required.

Variation problems

Variation problems

A **direct variation** is a special function that can be expressed as the equation $y = kx$, where k is a constant. The equation $y = kx$ is read "y varies directly as x" or "y is directly proportional to x."

The constant k is called the **constant of variation** or the **constant of proportionality.**

The circumference (C) of a circle varies directly as the diameter (d).

The direct variation equation is written $C = \pi d$. The constant of variation is π.

Variation problems

In general, a direct variation equation can be written in the form $y = kx^n$, where n is a positive number.

For example, the equation $y = kx^2$ is read "y varies directly as the square of x."

The direct variation equation can then be written by substituting the value of k into the basic direct variation equation.

Example 2

The amount (A) of medication prescribed for a person varies directly with the person's weight (W). For a person who weighs 50 kg, 2 ml of medication are prescribed. How many milliliters of medication are required for a person who weighs 75 kg?

Strategy:

To find the required amount of medication:

- Write the basic direct variation equation, replace the variables by the given values, and solve for k.
- Write the direct variation equation, replacing k by its value. Substitute 75 for W, and solve for A.

Example 2 – *Solution*

$$
\begin{aligned}
A &= kW \\
2 &= k \cdot 50 \\
\frac{1}{25} &= k \\
A &= \frac{1}{25}W && \textbf{This is the direct variation equation.} \\
A &= \frac{1}{25} \cdot 75 && \textbf{Replace } \boldsymbol{W} \textbf{ by 75.} \\
&= 3
\end{aligned}
$$

The required amount of medication is 3 ml.

Variation problems

An **inverse variation** is a function that can be expressed as the equation $y = \frac{k}{x}$, where *k* is a constant.

The equation $y = \frac{k}{x}$ is read "*y* varies inversely as *x*" or "*y* is inversely proportional to *x*."

In general, an inverse variation equation can be written $y = \frac{k}{x^n}$, where *n* is a positive number.

For example, the equation $y = \frac{k}{x^2}$ is read "*y* varies inversely as the square of *x*."

Variation problems

The inverse variation equation can then be found by substituting the value of k into the basic inverse variation equation.

Example 3

A company that produces personal computers has determined that the number of computers it can sell (s) is inversely proportional to the price (P) of the computer. Two thousand computers can be sold when the price is $900. How many computers can be sold when the price of a computer is $800?

Strategy:

To find the number of computers:

- Write the basic inverse variation equation, replace the variables by the given values, and solve for k.
- Write the inverse variation equation, replacing k by its value. Substitute 800 for P, and solve for s.

Example 3 – *Solution*

$$s = \frac{k}{P}$$

$$2000 = \frac{k}{900}$$

$$1{,}800{,}000 = k$$

$$s = \frac{1{,}800{,}000}{P}$$

This is the inverse variation equation.

$$s = \frac{1{,}800{,}000}{800}$$

Replace P by 800.

$$= 2250$$

At a price of $800, 2250 computers can be sold.

Variation problems

A **combined variation** is a variation in which two or more types of variation occur at the same time.

For example, in chemistry, the volume (*V*) of a gas varies directly as the temperature (*T*) and inversely as the pressure (*P*).

This combined variation is written $V = \dfrac{kT}{P}$.

A combined variation is the subject of Example 4.

Variation problems

A **joint variation** is a variation in which a variable varies directly as the product of two or more other variables.

A joint variation can be expressed as the equation $z = kxy$, where k is a constant. The equation $z = kxy$ is read "z varies jointly as x and y."

For example, the area (A) of a triangle varies jointly as the base (b) and the height (h). The joint variation equation is written $A = \frac{1}{2}bh$. The constant of variation is $\frac{1}{2}$.

Example 4

The pressure (P) of a gas varies directly as the temperature (T) and inversely as the volume (V). When $T = 50°$ and $V = 275$ in^3, $P = 20$ lb/in^2. Find the pressure of a gas when $T = 60°$ and $V = 250$ in^3.

Strategy:

To find the pressure:

- Write the basic combined variation equation, replace the variables by the given values, and solve for k.
- Write the combined variation equation, replacing k by its value. Substitute 60 for T and 250 for V, and solve for P.

Example 4 – *Solution*

$$P = \frac{kT}{V}$$

$$20 = \frac{k(50)}{275}$$

$$110 = k$$

$$P = \frac{110T}{V}$$

This is the combined variation equation.

$$P = \frac{110(60)}{250}$$

Replace *T* by 60 and *V* by 250.

$$= 26.4$$

The pressure is 26.4 lb/in^2.