CHAPTER 6

Factoring

Digital Vision

6.6 Solving Equations

Objectives

1 Solve equations by factoring

2 Application problems

Solve equations by factoring

Solve equations by factoring

We have know that the Multiplication Property of Zero states that the product of a number and zero is zero.

If a is a real number, then $a \cdot 0 = 0$.

Consider the equation $a \cdot b = 0$. If this is a true equation, then either $a = 0$ or $b = 0$.

Solve equations by factoring

PRINCIPLE OF ZERO PRODUCTS

If the product of two factors is zero, then at least one of the factors must be zero.

$$\text{If } \boldsymbol{a \cdot b = 0}, \text{ then } \boldsymbol{a = 0} \text{ or } \boldsymbol{b = 0}.$$

The Principle of Zero Products is used in solving equations.

Solve equations by factoring

An equation that can be written in the form $ax^2 + bx + c = 0$, $a \neq 0$, is a **quadratic equation.**

A quadratic equation is in **standard form** when the polynomial is equal to zero and its terms are in descending order.

A quadratic equation can be solved by using the Principle of Zero Products when the polynomial $ax^2 + bx + c$ is factorable.

Example 1

Solve: $2x^2 + x = 6$

Solution:

$$2x^2 + x = 6$$

This is a quadratic equation.

$$2x^2 + x - 6 = 0$$

Write it in standard form.

$$(2x - 3)(x + 2) = 0$$

Factor the trinomial.

$$2x - 3 = 0 \qquad x + 2 = 0$$

Set each factor equal to zero (the Principle of Zero Products).

$$2x = 3 \qquad x = -2$$

Solve each equation for *x*.

$$x = \frac{3}{2}$$

Example 1 – *Solution*

cont'd

Check:

$$
\begin{array}{r|l}
2x^2 + x = 6 & \\
\hline
2\left(\frac{3}{2}\right)^2 + \frac{3}{2} & 6 \\
2\left(\frac{9}{4}\right) + \frac{3}{2} & 6 \\
\frac{9}{2} + \frac{3}{2} & 6
\end{array}
$$

6 = 6

$$
\begin{array}{r|l}
2x^2 + x = 6 & \\
\hline
2(-2)^2 + (-2) & 6 \\
2 \cdot 4 - 2 & 6 \\
8 - 2 & 6
\end{array}
$$

6 = 6

The solutions are $\frac{3}{2}$ and –2.

Write the solutions.

Solve equations by factoring

Example 1 illustrates the steps involved in solving a quadratic equation by factoring.

STEPS IN SOLVING A QUADRATIC EQUATION BY FACTORING

1. Write the equation in standard form.
2. Factor the polynominal.
3. Set each factor equal to zero.
4. Solve each equation for the variable.
5. Check the solutions.

Application problems

Example 3

The sum of the squares of two consecutive positive odd integers is equal to 130. Find the two integers.

Strategy:

- First positive odd integer: n

  Second positive odd integer: $n + 2$

  Square of the first positive odd integer: n^2

  Square of the second positive odd integer: $(n + 2)^2$

- The sum of the square of the first positive odd integer and the square of the second positive odd integer is 130.

Example 3 – *Solution*

$n^2 + (n + 2)^2 = 130$	This is a quadratic equation.
$n^2 + n^2 + 4n + 4 = 130$	Square $n + 2$.
$2n^2 + 4n - 126 = 0$	Combine like terms. Subtract 130 from each side of the equation.
$2(n^2 + 2n - 63) = 0$	Factor out the common factor of 2.
$n^2 + 2n - 63 = 0$	Divide each side of the equation by 2.
$(n - 7)(n + 9) = 0$	Factor the trinomial.
$n - 7 = 0$ $\quad n + 9 = 0$	Set each factor equal to zero.
$n = 7$ $\quad n = -9$	Solve for *n.*

Example 3 – *Solution*

cont'd

Because –9 is not a positive odd integer, it is not a solution. The first odd integer is 7.

$n + 2 = 7 + 2 = 9$

Substitute the value of *n* into the variable expression for the second positive odd integer and evaluate.

The two integers are 7 and 9.