CHAPTER 6

Factoring

6.4 Special Factoring

Objectives

1. Factor the difference of two perfect squares and factor perfect-square trinomials

2. Factor the sum or the difference of two cubes

3. Factor trinomials that are quadratic in form

Factor the difference of two perfect squares and factor perfect-square trinomials

The product of a term and itself is called a **perfect square.** The exponents on variable parts of perfect squares are always even numbers.

The **square root of a perfect square** is one of the two equal factors of the perfect square. $\sqrt{}$ is the symbol for square root. To find the exponent of the square root of a variable term, divide the exponent by 2.

For the examples below, assume that *x* and *y* represent positive numbers.

$$\sqrt{25} = 5$$
$$\sqrt{x^2} = x$$
$$\sqrt{9y^8} = 3y^4$$

The factors of the difference of two perfect squares are the sum and difference of the square roots of the perfect squares.

FACTOR THE DIFFERENCE OF TWO SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

EXAMPLES

1. $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$
2. $z^2 - 49 = z^2 - 7^2 = (z + 7)(z - 7)$

An expression such as $a^2 + b^2$ is the *sum of two squares*. The sum of two squares does not factor over the integers.

For instance, $x^2 + 9$ is the sum of two squares. It does not factor over the integers.

Example 1

Factor. **A.** $25x^2 - 81$ **B.** $12x^3 - 147x$

Solution:

A. $25x^2 - 81 = (5x)^2 - 9^2$ — Write $25x^2 - 81$ as the difference of two squares.

$= (5x + 9)(5x - 9)$ — Use $a^2 - b^2 = (a + b)(a - b)$ to factor.

B. $12x^3 - 147x = 3x(4x^2 - 49)$ — Factor out the GCF.

$= 3x[(2x)^2 - 7^2]$ — Write $4x^2 - 49$ as the difference of two squares.

$= 3x(2x + 7)(2x - 7)$ — Use $a^2 - b^2 = (a + b)(a - b)$ to factor.

The square of a binomial is a **perfect-square trinomial.** Here are two examples.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

To factor a perfect-square trinomial, write the trinomial as the square of a binomial.

Factor the difference of two perfect squares and factor perfect-square trinomials

FACTOR A PERFECT-SQUARE TRINOMIAL

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

EXAMPLES

1. $x^2 + 6x + 9 = (x + 3)^2$
2. $x^2 - 12x + 36 = (x - 6)^2$

Example 2

Factor: $4x^2 - 20x + 25$

Solution:

$$4x^2 - 20x + 25$$ $4x^2 = (2x)^2$ and $25 = 5^2$.

$$= (2x - 5)^2$$ Check that $(2x - 5)^2 = 4x^2 - 20x + 25$.

Factor the sum or the difference of two cubes

Factor the sum or the difference of two cubes

The product of the same three factors is called a **perfect cube.**

The exponents on the variable parts of perfect cubes are always divisible by 3.

Term		*Perfect Cube*
2	$2 \cdot 2 \cdot 2 =$	8
$3y$	$3y \cdot 3y \cdot 3y =$	$27y^3$
x^2	$x^2 \cdot x^2 \cdot x^2 =$	x^6

Factor the sum or the difference of two cubes

The **cube root of a perfect cube** is one of the three equal factors of the perfect cube.

$\sqrt[3]{}$ is the symbol for cube root.

To find the exponent of the cube root of a perfect-cube variable term, divide the exponent by 3.

$$\sqrt[3]{8} = 2$$
$$\sqrt[3]{27y^3} = 3y$$
$$\sqrt[3]{x^6} = x^2$$

Factor the sum or the difference of two cubes

The following rules are used to factor the sum or difference of two perfect cubes.

FACTOR THE SUM OR DIFFERENCE OF TWO CUBES

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

EXAMPLES

1. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9)$

2. $z^3 - 64 = z^3 - 4^3 = (z - 4)(z^2 + 4z + 16)$

Example 3

Factor. **A.** $8x^3 + 125$ **B.** $3x^4y - 81xy^4$

Solution:

A. $8x^3 + 125 = (2x)^3 + 5^3$

$8x^3 + 125$ is the sum of two cubes.

$= (2x + 5)[(2x)^2 - 2x(5) + 5^2]$

Factor using the Sum of Two Cubes formula.

$= (2x + 5)(4x^2 - 10x + 25)$

Example 3 – *Solution*

cont'd

B. $3x^4y - 81xy^4$

$= 3xy(x^3 - 27y^3)$ — **$3xy$ is a common factor.**

$= 3xy[x^3 - (3y)^3]$ — **$x^3 - 27y^3$ is the difference of two cubes.**

$= 3xy(x - 3y)[(x)^2 + x(3y) + (3y)^2]$ — **Factor using the Difference of Two Cubes formula.**

$= 3xy(x - 3y)(x^2 + 3xy + 9y^2)$

Factor trinomials that are quadratic in form

Factor trinomials that are quadratic in form

Certain trinomials can be expressed as quadratic trinomials by making suitable variable substitutions.

TRINOMIALS THAT ARE QUADRATIC IN FORM

A trinomial is **quadratic in form** if it can be written as $au^2 + bu + c$.

EXAMPLES

1. $2x^6 - 7x^3 + 4$

Let $u = x^3$. Then $u^2 = (x^3)^2 = x^6$.

$2x^6 - 7x^3 + 4 \Rightarrow 2u^2 - 7u + 4$

$2x^6 - 7x^3 + 4$ is quadratic in form.

2. $5x^2y^2 + 3xy - 6$

Let $u = xy$. Then $u^2 = (xy)^2 = x^2y^2$.

$5x^2y^2 + 3xy - 6 \Rightarrow 5u^2 + 3u - 6$

$5x^2y^2 + 3xy - 6$ is quadratic in form.

Example 4

Factor. **A.** $6x^2y^2 - xy - 12$ **B.** $2x^4 + 5x^2 - 12$

Solution:

A. $6x^2y^2 - xy - 12$ Let $u = xy$.

$= (3xy + 4)(2xy - 3)$ Factor $6u^2 - u - 12$.

B. $2x^4 + 5x^2 - 12$ Let $u = x^2$.

$= (x^2 + 4)(2x^2 - 3)$ Factor $2u^2 + 5u - 12$.