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2 Factor completely



Trinomials of the form $x^2 + bx + c$, where *b* and *c* are integers, are shown below.

| $x^2 + 9x + 14$, | b = 9, | <mark>c</mark> = 14 |
|-------------------|-------------------------|----------------------|
| $x^2 - x - 12$, | <u></u> <i>b</i> = −1, | <mark>c</mark> = −12 |
| $x^2 - 2x - 15$, | <u></u> <i>b</i> = −2 , | <mark>c</mark> = −15 |

Some trinomials expressed as the product of binomials are shown at the right. They are in factored form.

| Trinomial | Factored Form |
|-------------------|----------------|
| $x^2 + 9x + 14 =$ | (x+2)(x+7) |
| $x^2 - x - 12 =$ | (x + 3)(x - 4) |
| $x^2 - 2x - 15 =$ | (x+3)(x-5) |

POINTS TO REMEMBER IN FACTORING $x^2 + bx + c$

- 1. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.
- 2. In the trinomial, the constant term is the product of the constant terms of the binomials.
- 3. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign as the coefficient of x in the trinomial.
- 4. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.

Example 1

Factor: $x^2 + 18x + 32$

Solution:

| Factors of 32 | Sum |
|-----------------------|-----------------|
| 1, 32 2, 16 | 33 18 |
| 4, 8 | 12 |

Try only positive factors of 32 [Point 3].

Once the correct pair is found, the other factors need not be tried.

 $x^2 + 18x + 32 = (x + 2)(x + 16)$ Write the factors of the trinomial.

Check:

$$(x + 2) (x + 16) = x^{2} + 16x + 2x + 32$$
$$= x^{2} + 18x + 32$$

Not all trinomials can be factored when using only integers. Consider the trinomial $x^2 - 6x - 8$.

| Factors of -8 | Sum |
|--|---------|
| $ \begin{array}{c} 1, -8 \\ -1, 8 \\ 2, -4 \end{array} $ | -7 7 -2 |
| -2, 4 | 2 |

Because none of the pairs of factors of -8 has a sum of -6, the trinomial is not factorable using integers.

The trinomial is said to be **nonfactorable over the integers**.



Factor completely



A polynomial is **factored completely** when it is written as a product of factors that are nonfactorable over the integers.

The first step in *any* factoring problem is to determine whether the terms of the polynomial have a *common factor*. If they do, factor it out first.



Factor: $3x^3 + 15x^2 + 18x$

Solution:

The GCF of $3x^3$, $15x^2$, and 18x is 3x.

Find the GCF of the terms of the polynomial.

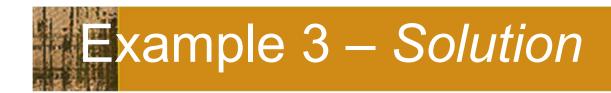
 $3x^3 + 15x^2 + 18x = 3x(x^2) + 3x(5x) + 3x(6)$ Factor out the GCF.

$$= 3x(x^2 + 5x + 6)$$

| Factors of 6 | Sum |
|--------------|-----|
| 1, 6 | 7 |
| 2, 3 | 5 |

Write the polynomial as a product of factors.

Factor the trinomial $x^2 + 5x + 6$. Try only positive factors of 6.



cont'd

 $3x^3 + 15x^2 + 18x = 3x(x + 2)(x + 3)$

Check:

 $3x(x+2) (x+3) = 3x(x^2 + 3x + 2x + 6)$

$$= 3x(x^2 + 5x + 6)$$

$$= 3x^3 + 15x^2 + 18x$$