CHAPTER 6

Factoring

6.1 Common Factors

Objectives

1 Factor a monomial from a polynomial

2 Factor by grouping

Factor a monomial from a polynomial

Factor a monomial from a polynomial

The **greatest common factor (GCF)** of two or more integers is the greatest integer that is a factor of all the integers.

The GCF of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

Note that the exponent of each variable in the GCF is the same as the *smallest* exponent of that variable in any of the monomials.

Example 1

Find the GCF of $12a^4b$ and $18a^2b^2c$.

Solution:

$12a^4b = 2 \cdot 2 \cdot 3 \cdot a^4 \cdot b$

Factor each monomial.

$18a^2b^2c = 2 \cdot 3 \cdot 3 \cdot a^2 \cdot b^2 \cdot c$

$\text{GCF} = 2 \cdot 3 \cdot a^2 \cdot b$

$= 6a^2b$

The common variable factors are a^2 and b. c is not a common factor.

The GCF of $12a^4b$ and $18a^2b^2c$ is $6a^2b$.

Factor a monomial from a polynomial

The Distributive Property is used to multiply factors of a polynomial. To **factor a polynomial** means to write the polynomial as a product of other polynomials.

Multiply

Factors		**Polynomial**
$2x(x + 5)$	$=$	$2x^2 + 10x$

Factor

In the example above, $2x$ is the GCF of the terms $2x^2$ and $10x$. It is a **common monomial factor** of the terms. $x + 5$ is a **binomial factor** of $2x^2 + 10x$.

Example 2

Factor. **A**. $5x^3 - 35x^2 + 10x$ **B**. $16x^2y + 8x^4y^2 - 12x^4y^5$

Solution:

A. The GCF is $5x$.

Find the GCF of the terms of the polynomial.

$$\frac{5x^3}{5x} = x^2, \frac{-35x^2}{5x} = -7x, \frac{10x}{5x} = 2$$

Divide each term of the polynomial by the GCF.

$5x^3 - 35x^2 + 10x$

$= 5x(x^2) + 5x(-7x) + 5x(2)$

Use the quotients to rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

Example 2 – *Solution*

cont'd

$= 5x(x^2 - 7x + 2)$

Use the Distributive Property to write the polynomial as a product of factors.

B. $16x^2y = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x^2 \cdot y$

Find the GCF of the terms of the polynomial.

$8x^4y^2 = 2 \cdot 2 \cdot 2 \cdot x^4 \cdot y^2$

$2x^4y^5 = 2 \cdot 2 \cdot 3 \cdot x^4 \cdot y^5$

The GCF is $4x^2y$.

Example 2 – *Solution*

cont'd

$$\frac{16x^2y}{4x^2y} = 4, \quad \frac{8x^4y^2}{4x^2y} = 2x^2y,$$

$$\frac{-12x^4y^5}{4x^2y} = -3x^2y^4$$

Divide each term of the polynomial by the GCF.

$16x^2y + 8x^4y^2 - 12x^4y^5$

$= 4x^2y(4) + 4x^2y(2x^2y) + 4x^2y(-3x^2y^4)$

Use the quotients to rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$= 4x^2y(4 + 2x^2y - 3x^2y^4)$

Use the Distributive Property to write the polynomial as a product of factors.

Factor by grouping

Factor by grouping

In the examples below, the binomials in parentheses are called **binomial factors.**

$2a(a + b)$

$3xy(x - y)$

The Distributive Property is used to factor a common binomial factor from an expression.

Example 3

Factor: $y(x + 2) + 3(x + 2)$

Solution:

$$y(x + 2) + 3(x + 2) = (x + 2)(y + 3)$$

The common binomial factor is $(x + 2)$

Factor by grouping

Sometimes a binomial factor must be rewritten before a common binomial factor can be found.

Example 4

Factor: $2x(x - 5) + y(5 - x)$

Solution:

$$2x(x - 5) + y(5 - x)$$

$$= 2x(x - 5) - y(x - 5)$$

Rewrite $5 - x$ as $-(x - 5)$ so that the terms have a common factor.

$$= (x - 5)(2x - y)$$

Write the expression as a product of factors.

Factor by grouping

Some polynomials can be factored by grouping the terms so that a common binomial factor is found.

Example 5

Factor. **A**. $2x^3 - 3x^2 + 8x - 12$ **B**. $3y^3 - 4y^2 - 6y + 8$

Solution:

A. $2x^3 - 3x^2 + 8x - 12$

$= (2x^3 - 3x^2) + (8x - 12)$

Group the first two terms and the last two terms.

$= x^2(2x - 3) + 4(2x - 3)$

Factor out the GCF from each group.

$= (2x - 3)(x^2 + 4)$

Factor out the common binomial factor and write the expression as a product of factors.

Example 5 – *Solution*

cont'd

B. $3y^3 - 4y^2 - 6y + 8$

$= (3y^3 - 4y^2) - (6y - 8)$

Group the first two terms and the last two terms. Note that $-6y + 8 = -(6y - 8)$.

$= y^2(3y - 4) - 2(3y - 4)$

Factor out the GCF from each group.

$= (3y - 4)(y^2 - 2)$

Factor out the common binomial factor and write the expression as a product of factors.