Polynomials

CHAPTER 5

5.5 Division of Polynomials

Objectives

1 Divide a polynomial by a monomial

2 Divide polynomials

3 Synthetic division

4 Evaluate a polynomial using synthetic division

Divide a polynomial by a monomial

Divide a polynomial by a monomial

To divide a polynomial by a monomial, divide each term in the numerator by the denominator, and write the sum of the quotients.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

Example 1

Divide: $\dfrac{6x^3 - 3x^2 + 9x}{3x}$

Solution:

$$\frac{6x^3 - 3x^2 + 9x}{3x} = \frac{6x^3}{3x} - \frac{3x^2}{3x} + \frac{9x}{3x}$$

Divide each term of the polynomial by the monomial 3*x*.

$$= 2x^2 - x + 3$$

Simplify each expression.

Divide polynomials

Divide polynomials

To divide polynomials, use a method similar to that used for division of whole numbers. The same equation used to check division of whole numbers is used to check polynomial division.

(Quotient × Divisor) + Remainder = Dividend

If a term is missing in the dividend, insert the term with zero as its coefficient. This helps keep like terms in the same column. This is illustrated in Example 3.

Example 3

Divide: $(6x + 2x^3 + 26) \div (x + 2)$

Solution:

$$\begin{array}{r} 2x^2 - 4x + 14 \\ x + 2\overline{)2x^3 + 0x^2 + 6x + 26} \\ \underline{2x^3 + 4x^2} \qquad\qquad \\ -4x^2 + 6x \qquad \\ \underline{-4x^2 - 8x} \qquad \\ 14x + 26 \\ \underline{14x + 28} \\ -2 \end{array}$$

Arrange the terms in descending order. There is no x^2 term in $2x^3 + 6x + 26$. Insert $0x^2$ for the missing term so that like terms will be in the same columns.

$$(6x + 2x^3 + 26) \div (x + 2) = 2x^2 - 4x + 14 - \frac{2}{x + 2}$$

Synthetic division

Synthetic division

Synthetic division is a shorter method of dividing a polynomial by a binomial of the form *x* – *a*. This method of dividing uses only the coefficients of the variable terms and the constant term.

Both long division and synthetic division are used below to simplify the expression

$$(3x^2 - 10x + 7) \div (x - 2).$$

Synthetic division

Long Division:

Compare the coefficients in this problem worked by long division with the coefficients in the same problem worked by synthetic division.

$$
\begin{array}{r}
3x - 4 \\
x - 2 \overline{)3x^2 - 10x + 7} \\
\underline{3x^2 - 6x} \\
-4x + 7 \\
\underline{-4x + 8} \\
-1
\end{array}
$$

$$(3x^2 - 10x + 7) \div (x - 2) = 3x - 4 - \frac{1}{x - 2}$$

Synthetic division

Synthetic Division:

Identify the value of *a*.
$x - a = x - 2$, so $a = 2$.
Bring down the 3.

Value of a Coefficients of the dividend

$$\begin{array}{r|rrr} 2 & 3 & -10 & 7 \\ & \downarrow & & \\ \hline & 3 & & \end{array}$$

Multiply $2 \cdot 3$ and add the product (6) to -10.
The result is -4.

$$\begin{array}{r|rrr} 2 & 3 & -10 & 7 \\ & & 6 & \\ \hline & 3 & -4 & \end{array}$$

Multiply $2(-4)$ and add the product (-8) to 7.
The result is -1.

$$\begin{array}{r|rrr} 2 & 3 & -10 & 7 \\ & & 6 & -8 \\ \hline & 3 & -4 & -1 \\ & & & \uparrow \end{array}$$

Coefficients of the quotient   Remainder

Synthetic division

Use the coefficients of the quotient and the remainder to write the result of the division. The degree of the first term of the quotient is one degree less than the degree of the first term of the dividend.

$$(3x^2 - 10x + 7) \div (x - 2)$$

$$= 3x - 4 - \frac{1}{x - 2}$$

Check: $(3x - 4)(x - 2) - 1 = 3x^2 - 6x - 4x + 8 - 1 = 3x^2 - 10x + 7$

Example 4

Divide.

A. $(5x^2 - 3x + 7) \div (x - 1)$

B. $(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$

Solution:

A.
$$\begin{array}{r|rrr} 1 & 5 & -3 & 7 \\ & & 5 & 2 \\ \hline & 5 & 2 & 9 \end{array}$$

$x - a = x - 1; a = 1$

$$(5x^2 - 3x + 7) \div (x - 1)$$

$$= 5x + 2 + \frac{9}{x - 1}$$

Example 4 – *Solution*

cont'd

B.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -8 & 2 & 1 \\ & & -6 & 12 & -8 & 12 \\ \hline & 3 & -6 & 4 & -6 & 13 \end{array}$$

Insert a zero for the missing x^3 term.

$x - a = x + 2;\ a = -2$

$$(3x^4 - 8x^2 + 2x + 1) \div (x + 2)$$

$$= 3x^3 - 6x^2 + 4x - 6 + \frac{13}{x + 2}$$

Evaluate a polynomial using synthetic division

Evaluate a polynomial using synthetic division

A polynomial can be evaluated by using synthetic division.

REMAINDER THEOREM

If the polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.

Example 5

Use the Remainder Theorem to evaluate $P(-2)$ when $P(x) = x^3 - 3x^2 + x + 3$.

Solution:

Use synthetic division with $a = -2$.

$$\begin{array}{r|rrrr} -2 & 1 & -3 & 1 & 3 \\ & & -2 & 10 & -22 \\ \hline & 1 & -5 & 11 & -19 \end{array}$$

By the Remainder Theorem, $P(-2) = -19$.