CHAPTER 5

Polynomials

5.4 Integer Exponents and Scientific Notation

Objectives

1 Integer exponents

2 Scientific notation

Integer exponents

Integer exponents

The quotient of two exponential expressions with the *same* base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

Note that subtracting the exponents results in the same quotient.

To divide two monomials with the same base, subtract the exponents of the like bases.

Integer exponents

For any number a, $a \neq 0$, $\frac{a}{a} = 1$. This property is true for exponential expressions as well. For example, for $x \neq 0$, $\frac{x^4}{x^4} = 1$.

This expression also can be simplified using the rule for dividing exponential expressions with the same base.

$$\frac{x^4}{x^4} = x^{4-4} = x^0$$

Because $\frac{x^4}{x^4} = 1$ and $\frac{x^4}{x^4} = x^0$, the following definition of zero as an exponent is used.

Integer exponents

ZERO AS AN EXPONENT

If $\boldsymbol{x} \neq \mathbf{0}$, then $\boldsymbol{x}^{\mathbf{0}} = \mathbf{1}$. The expression $\mathbf{0}^{\mathbf{0}}$ is not defined.

EXAMPLES

1. Simplify: $(12a^3)^0$, $a \neq 0$

Any nonzero expression to the zero power is 1. $(12a^3)^0 = 1$

2. Simplify: $-(y^4)^0$, $y \neq 0$

Any nonzero expression to the zero power is 1.

Because the negative sign is outside the parentheses, the answer is -1. $-(y^4)^0 = -1$

Integer exponents

DEFINITION OF NEGATIVE EXPONENTS

If n is a positive integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

EXAMPLES

In each example below, simplify the expression by writing it with a positive exponent.

1. $x^{-10} = \dfrac{1}{x^{10}}$ **2.** $\dfrac{1}{a^{-5}} = a^5$ **3.** $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

Integer exponents

Now that negative exponents have been defined, the Rule for Dividing Exponential Expressions can be stated.

RULE FOR DIVIDING EXPONENTIAL EXPRESSIONS

If m and n are integers and $\boldsymbol{x \neq 0}$, then $\boldsymbol{\frac{x^m}{x^n} = x^{m-n}}$.

EXAMPLES

Simplify each expression below by using the Rule for Dividing Exponential Expressions.

1. $\frac{x^3}{x^5} = x^{3-5} = x^{-2} = \frac{1}{x^2}$

2. $\frac{y^6}{y^{-2}} = y^{6-(-2)} = y^8$

3. $\frac{b^{-5}}{b^{-1}} = b^{-5-(-1)} = b^{-4} = \frac{1}{b^4}$

4. $\frac{a^{-4}}{a^{-7}} = a^{-4-(-7)} = a^3$

Example 1

Write $\frac{3^{-3}}{3^2}$ with a positive exponent. Then evaluate.

Solution:

$$\frac{3^{-3}}{3^2} = 3^{-3-2}$$

3^{-3} and 3^2 have the same base. Subtract the exponents.

$$= 3^{-5}$$

$$= \frac{1}{3^5}$$

Use the Definition of Negative Exponents to write the expression with a positive exponent.

$$= \frac{1}{243}$$

Evaluate.

Integer exponents

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integers. These rules are restated here.

RULES FOR EXPONENTS

If m, n, and p are integers, then

$x^m \cdot x^n = x^{m+n}$ $\qquad (x^m)^n = x^{mn}$ $\qquad (x^m y^n)^p = x^{mp} y^{np}$

$\dfrac{x^m}{x^n} = x^{m-n}, x \neq 0$ $\qquad x^{-n} = \dfrac{1}{x^n}, x \neq 0$ $\qquad x^0 = 1, x \neq 0$

An exponential expression is in simplest form when it is written with only positive exponents.

Example 2

Simplify: **A.** $a^{-7}b^3$ **B.** $\dfrac{x^{-4}y^6}{xy^2}$ **C.** $6d^{-4}, d \neq 0$

Solution:

A. $a^{-7}b^3 = \dfrac{b^3}{a^7}$

Rewrite a^{-7} with a positive exponent.

B. $\dfrac{x^{-4}y^6}{xy^2} = x^{-4-1}y^{6-2}$

Divide variables with the same base by subtracting the exponents.

$= x^{-5}y^4$

$= \dfrac{y^4}{x^5}$

Write the expression with only positive exponents.

Example 2 – *Solution*

cont'd

c. $6d^{-4} = 6 \cdot \frac{1}{d^4}$

Use the Definition of Negative Exponents to rewrite the expression with a positive exponent.

$= \frac{6}{d^4}$

Scientific notation

Scientific notation

Very large and very small numbers are encountered in the fields of science and engineering. For example, the charge of an electron is 0.000000000000000000160 coulomb. These numbers can be written more easily in scientific notation. In **scientific notation,** a number is expressed as a product of two factors, one a number between 1 and 10 and the other a power of 10.

To change a number written in decimal notation to scientific notation, write it in the form $a \times 10^n$, where *a* is a number between 1 and 10 and *n* is an integer.

Example 4

Write the number in scientific notation.

A. 824,300,000,000 **B.** 0.000000961

Solution:

A. 824,300,000,000 = 8.243×10^{11}

Move the decimal point 11 places to the left. The exponent on 10 is 11.

B. 0.000000961 = 9.61×10^{-7}

Move the decimal point 7 places to the right. The exponent on 10 is –7.

Scientific notation

Changing a number written in scientific notation to decimal notation also requires moving the decimal point.

When the exponent on 10 is positive, move the decimal point to the right the same number of places as the exponent.

$$3.45 \times 10^9 = 3{,}450{,}000{,}000$$

$$2.3 \times 10^8 = 230{,}000{,}000$$

When the exponent on 10 is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$$8.1 \times 10^{-3} = 0.0081$$

$$6.34 \times 10^{-6} = 0.00000634$$

Example 5

Write the number in decimal notation.

A. 7.329×10^6 **B.** 6.8×10^{-10}

Solution:

A. $7.329 \times 10^6 = 7{,}329{,}000$

The exponent on 10 is positive. Move the decimal point 6 places to the right.

B. $6.8 \times 10^{-10} = 0.00000000068$

The exponent on 10 is negative. Move the decimal point 10 places to the left.

Scientific notation

The rules for multiplying and dividing with numbers in scientific notation are the same as those for calculating with algebraic expressions. The power of 10 corresponds to the variable, and the number between 1 and 10 corresponds to the coefficient of the variable.

Example 6

Multiply or divide.

A. $(3.0 \times 10^{5})(1.1 \times 10^{-8})$

B. $\dfrac{7.2 \times 10^{13}}{2.4 \times 10^{-3}}$

Solution:

A. $(3.0 \times 10^{5})(1.1 \times 10^{-8}) = 3.3 \times 10^{-3}$

Multiply 3.0 and 1.1.
Add the exponents on 10.

B. $\dfrac{7.2 \times 10^{13}}{2.4 \times 10^{-3}} = 3 \times 10^{16}$

Divide 7.2 by 2.4.
Subtract the exponents on 10.