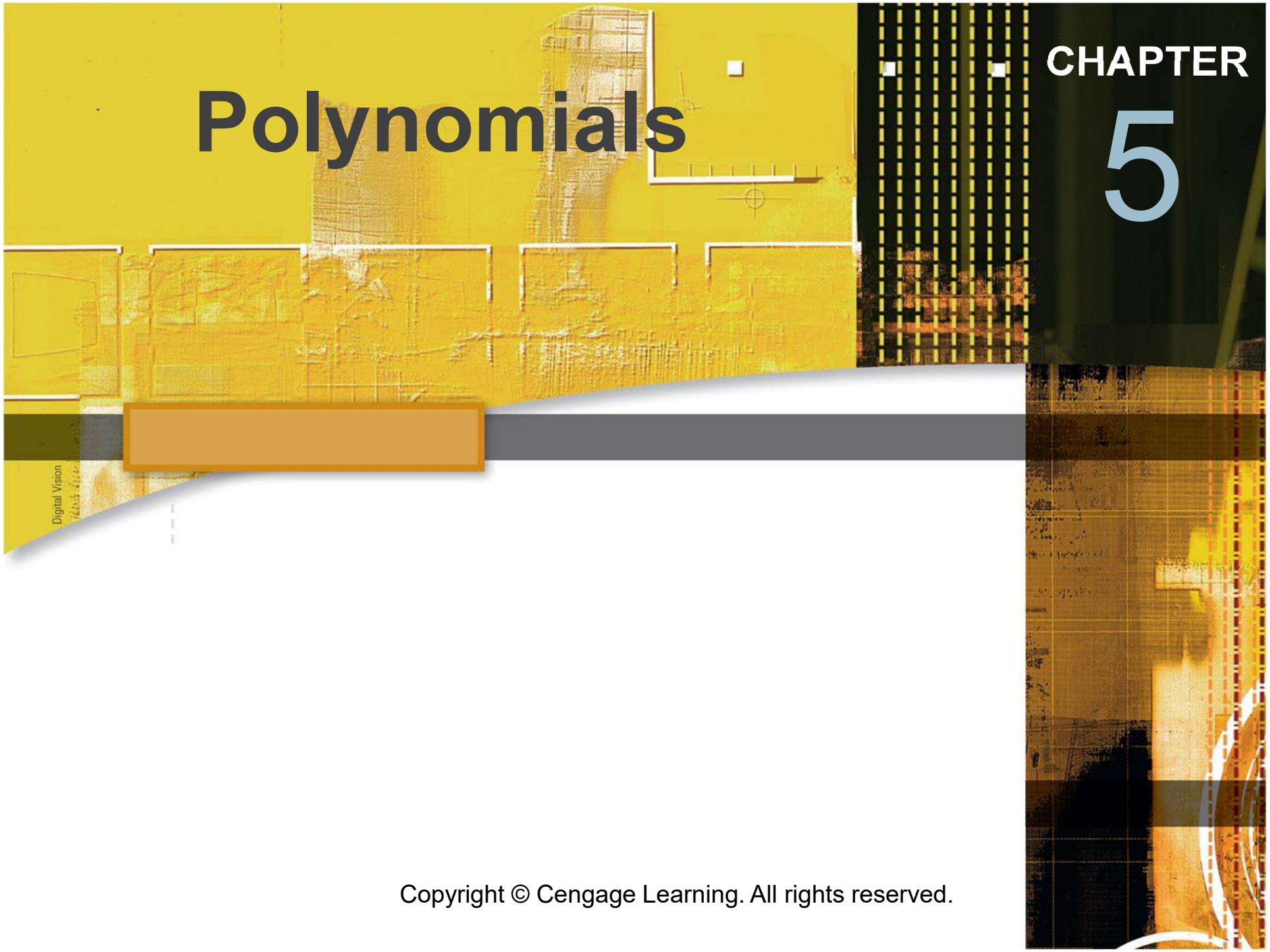
Polynomials

CHAPTER 5



5.2 Multiplication of Monomials

Objectives

1. Multiply monomials
2. Simplify powers of monomials

Multiply monomials

Multiply monomials

In the exponential expression x^5, x is the base and 5 is the exponent. The exponent indicates the number of times the base occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^2 = \overbrace{(x \cdot x \cdot x)}^{3 \text{ factors}} \cdot \overbrace{(x \cdot x)}^{2 \text{ factors}}$$

$$= \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{5 \text{ factors}}$$

$$= x^5$$

Multiply monomials

Adding the exponents results in the same product.

$$x^3 \cdot x^2 = x^{3+2}$$

$$= x^5$$

RULE FOR MULTIPLYING EXPONENTIAL EXPRESSIONS

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

EXAMPLES

In each example below, we are multiplying two exponential expressions with the same base. Simplify the expression by adding the exponents.

1. $x^4 \cdot x^7 = x^{4+7} = x^{11}$
2. $y \cdot y^5 = y^{1+5} = y^6$
3. $a^2 \cdot a^6 \cdot a = a^{2+6+1} = a^9$

Example 1

Multiply: $(2xy)(3x^2y)$

Solution:

$(2xy)\,(3x^2y) = (2 \cdot 3)(x \cdot x^2)(y \cdot y)$

Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

$= 6x^{1+2}\,y^{1+1}$

Multiply variables with the same base by adding the exponents.

$= 6x^3y^2$

Simplify powers of monomials

Simplify powers of monomials

A power of a monomial can be simplified by rewriting the expression in factored form and then using the Rule for Multiplying Exponential Expressions.

RULE FOR SIMPLIFYING POWERS OF EXPONENTIAL EXPRESSIONS

If m and n are integers, then $(x^m)^n = x^{mn}$.

EXAMPLES

Each example below is a power of an exponential expression. Simplify the expression by multiplying the exponents.

1. $(x^5)^2 = x^{5 \cdot 2} = x^{10}$
2. $(y^3)^4 = y^{3 \cdot 4} = y^{12}$

Simplify powers of monomials

RULE FOR SIMPLIFYING POWERS OF PRODUCTS

If m, n, and p are integers, then $(x^m y^n)^p = x^{mp} y^{np}$.

EXAMPLES

Each example below is a power of a product of exponential expressions. Simplify the expression by multiplying each exponent inside the parentheses by the exponent outside the parentheses.

1. $(c^5 d^3)^6 = c^{5 \cdot 6} d^{3 \cdot 6} = c^{30} d^{18}$

2. $(3a^2 b)^3 = 3^{1 \cdot 3} a^{2 \cdot 3} b^{1 \cdot 3} = 3^3 a^6 b^3 = 27a^6 b^3$

Example 3

Simplify: $(-2x)(-3xy^2)^3$

Solution:

$(-2x)(-3xy^2)^3 = (-2x)(-3)^3\ x^3y^6$

Multiply each exponent in $-3xy^2$ by the exponent outside the parentheses.

$= (-2x)(-27)\ x^3y^6$

Simplify $(-3)^3$.

$= [-2(-27)](x \cdot x^3)y^6$

Use the Properties of Multiplication to rearrange and group factors.

$= 54x^4y^6$

Multiply variable expressions with the same base by adding the exponents.