# Polynomials

CHAPTER 5

# 5.1 Introduction to Polynomials

# Objectives

1. Evaluate polynomial functions

2. Add or subtract polynomials

# Evaluate polynomial functions

# Evaluate polynomial functions

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of one term is a **monomial.** $5x$

A polynomial of two terms is a **binomial.** $5x^2y + 6x$

A polynomial of three terms is a **trinomial.** $3x^2 + 9xy - 5y$

# Evaluate polynomial functions

The **degree of a polynomial** is the greatest of the degrees of any of its terms.

The terms of a polynomial in one variable are usually arranged so that the exponents on the variable decrease from left to right. This is called **descending order.**

The **linear function** given by $f(x) = mx + b$ is an example of a polynomial function. It is a polynomial function of degree 1. A second-degree polynomial function, called a **quadratic function**, is given by the equation $f(x) = ax^2 + bx + c, a \neq 0.$

# Evaluate polynomial functions

A third-degree polynomial function is called a **cubic function.**

In general, a **polynomial function** is an expression whose terms are monomials.

The **leading coefficient** of a polynomial function is the coefficient of the term with the greatest exponent on a variable.

The constant term is the term without a variable.

# Example 1

Find the leading coefficient, the constant term, and the degree of the polynomial $P(x) = 5x^6 - 4x^5 - 3x^2 + 7$.

Solution:

The term with the greatest exponent is $5x^6$. The term without a variable is 7.

The leading coefficient is 5, the constant term is 7, and the degree is 6.

# Evaluate polynomial functions

To **evaluate a polynomial function,** replace the variable by its value and simplify.

# Example 2

To overcome the resistance of the wind and the tires on the road, the horsepower (hp), *P*, required by a cyclist to keep a certain bicycle moving at *v* miles per hour is given by $P(v) = 0.00003v^3 + 0.00211v$. How much horsepower must the cyclist supply to keep this bicycle moving at 20 mph?

Strategy:

To find the horsepower, evaluate the function when $v$ = 20.

Solution:

$$P(v) = 0.00003v^3 + 0.00211v$$

# Example 2 – *Solution*

cont'd

$$P(20) = 0.00003(20)^3 + 0.00211(20)$$ **Replace $v$ by 20.**

$$= 0.2822$$ **Simplify.**

The cyclist must supply 0.2822 hp.

# Evaluate polynomial functions

The graph of a linear function is a straight line and can be found by plotting just two points. The graph of a polynomial function of degree greater than 1 is a curve. Consequently, many points may have to be found before an accurate graph can be drawn.

We have known that the domain of a function is the set of first coordinates of the ordered pairs of the function.

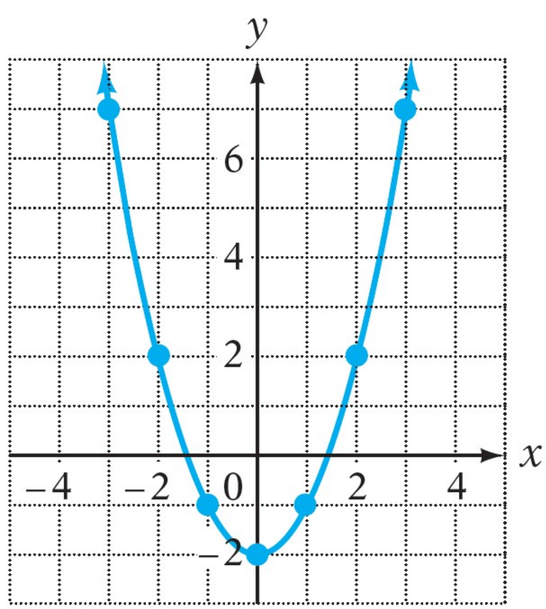
Because a polynomial function can be evaluated for any real number, the domain of a polynomial function is the set of real numbers.

# Example 3

Graph: $f(x) = x^2 - 2$

Solution:

| $x$ | $y = x^2 - 2$ |
|---|---|
| $-3$ | $7$ |
| $-2$ | $2$ |
| $-1$ | $-1$ |
| $0$ | $-2$ |
| $1$ | $-1$ |
| $2$ | $2$ |
| $3$ | $7$ |

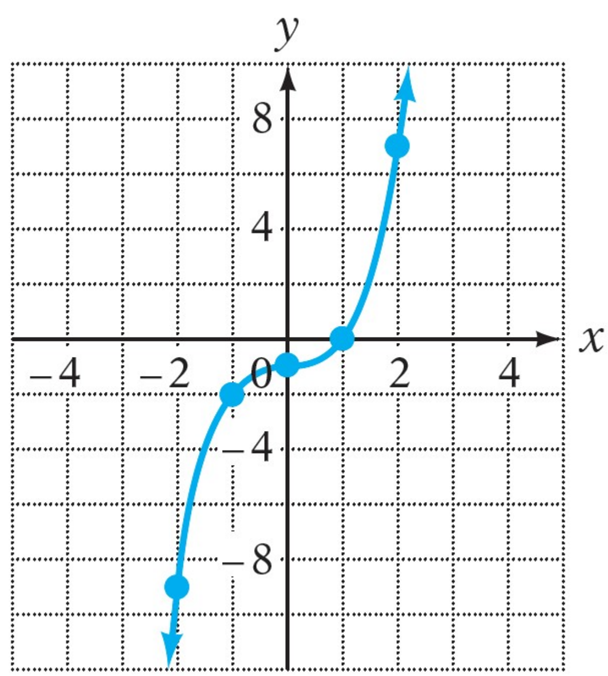


# Example 4

Graph: $f(x) = x^3 - 1$

Solution:

| $x$ | $y = x^3 - 1$ |
|---|---|
| $-2$ | $-9$ |
| $-1$ | $-2$ |
| $0$ | $-1$ |
| $1$ | $0$ |
| $2$ | $7$ |



## Add or subtract polynomials

# Add or subtract polynomials

Polynomials can be added by combining like terms. Either a vertical or a horizontal format can be used.

# Example 5

Add $(2x^3 + 5x^2 - 7x + 1) + (-x^3 - 5x^2 + 3x - 6)$ using a vertical format.

Solution:

$$\begin{array}{r} 2x^3 + 5x^2 - 7x + 1 \\ -x^3 - 5x^2 + 3x - 6 \\ \hline \end{array}$$

**Write each polynomial in descending order with like terms in columns.**

$$x^3 + 0x^2 - 4x - 5$$

**Add the terms in each column.**

$$(2x^3 + 5x^2 - 7x + 1) + (-x^3 - 5x^2 + 3x - 6) = x^3 - 4x - 5$$

# Add or subtract polynomials

The additive inverse of the polynomial $x^2 + 5x - 4$ is $-(x^2 + 5x - 4)$.

To simplify the additive inverse of a polynomial, change the sign of every term inside the parentheses.

$$-(x^2 + 5x - 4) = -x^2 - 5x + 4$$

To subtract two polynomials, add the additive inverse of the second polynomial to the first.

# Example 6

Subtract $(3x^2 - 2x + 4) - (7x^2 + 3x - 12)$ using a vertical format.

Solution:

$$(3x^2 - 2x + 4) - (7x^2 + 3x - 12)$$
$$(3x^2 - 2x + 4) + (-7x^2 - 3x + 12)$$

**Rewrite the subtraction as addition of the additive inverse.**

$$\begin{array}{r} 3x^2 - 2x + \phantom{1}4 \\ -7x^2 - 3x + 12 \\ \hline \end{array}$$

**Write each polynomial in descending order in a vertical format.**

$$-4x^2 - 5x + 16$$

**Combine like terms in each column.**