CHAPTER 4

Systems of Equations and Inequalities

4.1 Solving Systems of Linear Equations by Graphing and by the Substitution Method

Objectives

1 Solve a system of linear equations by graphing

2 Solve a system of linear equations by the substitution method

Solve a system of linear equations by graphing

Solve a system of linear equations by graphing

A **system of equations** is two or more equations considered together. The system at the right is a system of two linear equations in two variables.

The graphs of the equations are straight lines.

$$\begin{aligned} 3x + 4y &= 7 \\ 2x - 3y &= 6 \end{aligned}$$

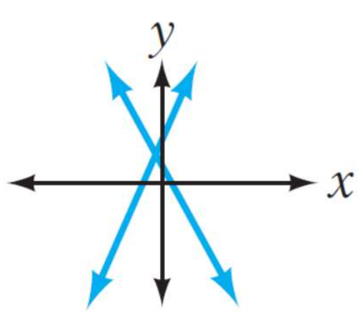
A **solution of a system of equations in two variables** is an ordered pair that is a solution of each equation of the system.

Solve a system of linear equations by graphing

A solution of a system of linear equations can be found by graphing the equations of the system on the same set of coordinate axes.

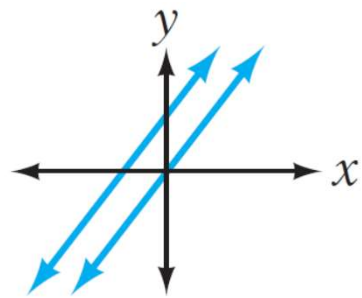
The three possibilities for a system of linear equations in two variables are:

1. The graphs intersect at one point. The solution of the system of equations is the ordered pair (x, y) whose coordinates are the point of intersection. The system of equations is *independent*.

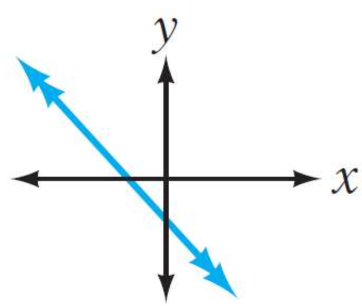


Solve a system of linear equations by graphing

2. The lines are parallel and never intersect. The system of equations has no solution. The system of equations is *inconsistent*.



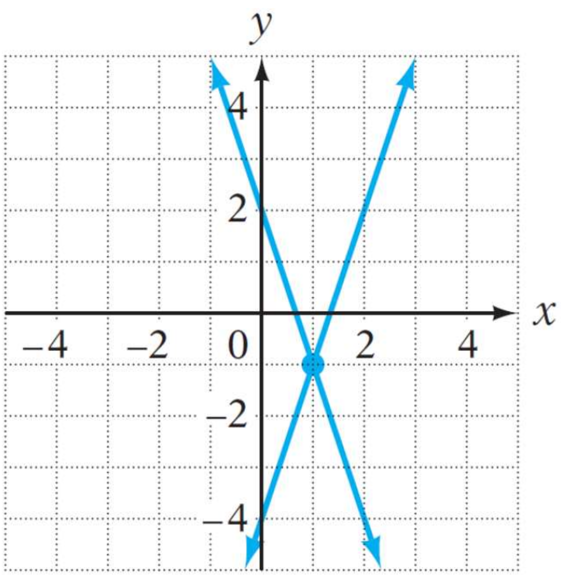
3. The graphs are the same line, and they intersect at infinitely many points. There are an infinite number of solutions of the system of equations. The system of equations is *dependent*.



Example 1

Solve by graphing: $\begin{aligned} 3x - y &= 4 \\ 3x + y &= 2 \end{aligned}$

Solution:



Graph each line. The graphs intersect. The coordinates of the point of intersection give the ordered-pair solution of the system.

The solution is (1, –1).

Example 2

Solve by graphing.

A. $2x + 3y = 6$

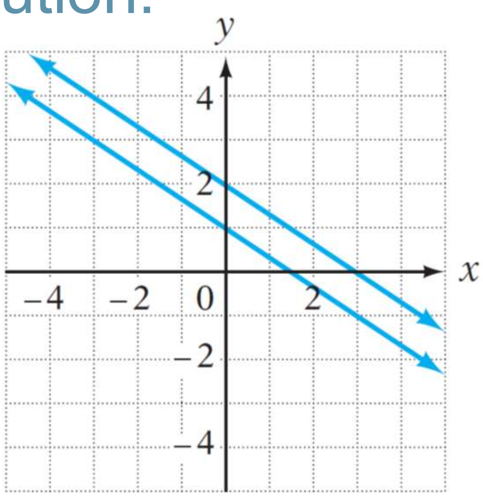
$$y = -\frac{2}{3}x + 1$$

B. $x - 2y = 6$

$$y = \frac{1}{2}x - 3$$

Solution:

A.



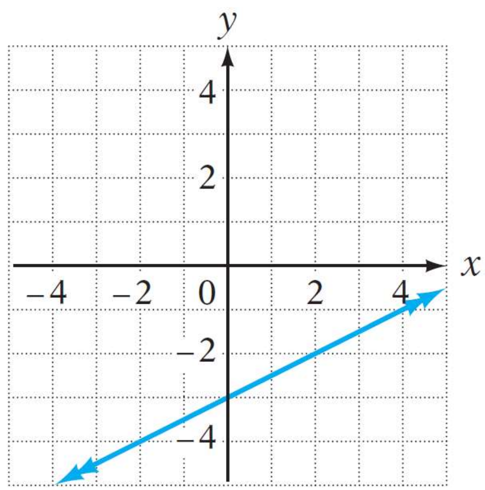
Graph each line. The graphs are parallel. The system of equations is inconsistent.

The system of equations has no solution.

Example 2 – *Solution*

cont'd

B.



Graph each line. The graphs are the same line. The system of equations is dependent.

For this system of equations, one of the equations is already solved for *y*.

The solutions are the ordered pairs $\left(x, \frac{1}{2}x - 3\right)$.

Solve a system of linear equations by the substitution method

Solve a system of linear equations by the substitution method

Solving a system of equations by graphing is based on approximating the coordinates of a point of intersection.

An algebraic method called the **substitution method** can be used to find an *exact* solution of a system of equations. To use the substitution method, we must write one of the equations of the system in terms of x or in terms of y.

Example 3

Solve by substitution.

A. $$\begin{aligned} 3x - 5y &= 9 \\ x &= 2y + 4 \end{aligned}$$

B. $$\begin{aligned} 3x - 3y &= 2 \\ y &= x + 2 \end{aligned}$$

C. $$\begin{aligned} 9x + 3y &= 12 \\ y &= -3x + 4 \end{aligned}$$

Example 3(a) – *Solution*

(1) $$3x - 5y = 9$$

(2) $$x = 2y + 4$$

$$3(2y + 4) - 5y = 9$$ Substitute $2y + 4$ for x in equation (1).

$$6y + 12 - 5y = 9$$ Solve for y.

$$y + 12 = 9$$

$$y = -3$$

$$x = 2y + 4$$ Use equation (2).

$$x = 2(-3) + 4$$ Substitute -3 for y.

$$= -2$$ Simplify.

The solution is $(-2, -3)$.

Example 3(b) – *Solution*

cont'd

(1)	$3x - 3y = 2$	
(2)	$y = x + 2$	**Equation (2) states that $y = x + 2$.**
	$3x - 3(x + 2) = 2$	**Substitute $x + 2$ for y in equation (1).**
	$3x - 3x - 6 = 2$	**Solve for x.**
	$-6 = 2$	

–6 = 2 is not a true equation. The system of equations is inconsistent. The system has no solution.

Example 3(c) – *Solution*

cont'd

(1) $$9x + 3y = 12$$

(2) $$y = -3x + 4$$

Equation (2) states that $y = -3x + 4$.

$$9x + 3(-3x + 4) = 12$$

Substitute $-3x + 4$ for y.

$$9x - 9x + 12 = 12$$

Solve for x.

$$12 = 12$$

12 = 12 is a true equation. The system of equations is dependent.

The solutions are the ordered pairs (x, $-3x + 4$).