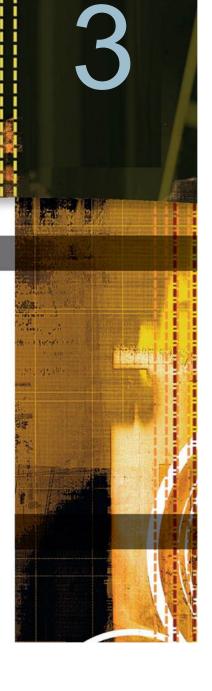
Linear Functions and Inequalities in Two Variables

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CHAPTER



Finding Equations of Lines

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Find the equation of a line given a point and the slope



When the slope of a line and a point on the line are known, the equation of the line can be determined. If the particular point is the *y*-intercept, use the slope-intercept form, y = mx + b, to find the equation.

One method of finding the equation of a line when the slope and any point on the line are known involves using the *point-slope formula*. This formula is derived from the formula for the slope of a line.

Find the equation of a line given a point and the slope

Let $P_1(x_1, y_1)$ be the given point on the line, and let P(x, y) be another point on the line.

Use the formula for the slope of a line.

$$\frac{y - y_1}{x - x_1} = m$$

Multiply each side of the equation by $(x - x_1)$.

$$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$$

Then simplify.

$$y-y_1=m(x-x_1)$$

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Find the equation of a line given a point and the slope

POINT-SLOPE FORMULA

Let *m* be the slope of a line, and let $P_1(x_1, y_1)$ be a point on the line. The equation of the line can be found by using the **point-slope formula**:

 $y - y_1 = m(x - x_1)$

Example 1

Find the equation of the line that contains the point P(-2, 4) and has slope 2.

Solution:

$$y - y_1 = m(x - x_1)$$

Use the point-slope formula.

$$y - 4 = 2[x - (-2)]$$

Substitute the slope, 2, and the coordinates of the given point, (-2, 4), into the point-slope formula.

$$y - 4 = 2(x + 2)$$
 Solve for y.



cont'd

$$y-4=2x+4$$

$$y = 2x + 8$$

The equation of the line is y = 2x + 8.



Find the equation of a line given two points



The point-slope formula and the formula for slope are used to find the equation of a line when two points are known.



Find the equation of the line containing the points $P_1(2, 3)$ and $P_2(4, 1)$.

 y_1 = (2, 3) and

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 2}$$

Find the slope. Let
 $(x_1, y_1) = (2, 3)$ and
 $(x_2, y_2) = (4, 1).$
 $= -1$



cont'd

$$y - y_1 = m(x - x_1)$$

$$y-3=-1(x-2)$$

Substitute the slope and the coordinates of either one of the known points into the point-slope formula.

$$y - 3 = -x + 2$$

Solve for y.

y = -x + 5

The equation of the line is y = -x + 5.



Application problems

Application problems

Linear functions can be used to model a variety of applications in science and business. For each application, data are collected and the independent and dependent variables are selected. Then a linear function is that models the data is determined.



In 2000, there were approximately 50,000 centenarians (people 100 years old or older). Data from the Census Bureau show that this population is expected to increase through the year 2020 at a rate of approximately 4250 centenarians per year. Find a linear function that approximates the population of centenarians in terms of the year. Use your function to approximate the number of centenarians in 2015.





Strategy:

Select the independent and dependent variables. Because we want to determine the population of centenarians, that quantity is the *dependent* variable, *y*. The year is the *independent* variable.

From the data, the ordered pair (2000, 50,000) gives the coordinates of a point on the line. The slope of the line is the *rate of increase*, 4250 centenarians per year.



$$y - y_1 = m(x - x_1)$$

Use the point-slope formula.

$$y - 50,000 = 4250(x - 2000)$$

m = 4250; $(x_1, y_1) = (2000, 50,000)$

$$y - 50,000 = 4250x - 8,500,000$$

$$y = 4250x - 8,450,000$$

The linear function is f(x) = 4250x - 8,450,000.



$$f(x) = 4250x - 8,450,000$$

f(2015) = 4250(2015) - 8,450,000

Evaluate the function at 2015 to predict the number of centenarians in 2015.

= 8,563,750 - 8,450,000

= 113,750

The function gives an estimate of 113,750 centenarians in 2015.