CHAPTER 3

Linear Functions and Inequalities in Two Variables

3.5 Finding Equations of Lines

Objectives

1. Find the equation of a line given a point and the slope
2. Find the equation of a line given two points
3. Application problems

Find the equation of a line given a point and the slope

Find the equation of a line given a point and the slope

When the slope of a line and a point on the line are known, the equation of the line can be determined. If the particular point is the y-intercept, use the slope-intercept form, $y = mx + b$, to find the equation.

One method of finding the equation of a line when the slope and any point on the line are known involves using the *point-slope formula*. This formula is derived from the formula for the slope of a line.

Find the equation of a line given a point and the slope

Let $P_1(x_1, y_1)$ be the given point on the line, and let $P(x, y)$ be another point on the line.

Use the formula for the slope of a line.

$$\frac{y - y_1}{x - x_1} = m$$

Multiply each side of the equation by $(x - x_1)$.

$$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$$

Then simplify.

$$y - y_1 = m(x - x_1)$$

Find the equation of a line given a point and the slope

POINT-SLOPE FORMULA

Let m be the slope of a line, and let $P_1(x_1, y_1)$ be a point on the line. The equation of the line can be found by using the **point-slope formula:**

$$y - y_1 = m(x - x_1)$$

Example 1

Find the equation of the line that contains the point $P(-2, 4)$ and has slope 2.

Solution:

$$y - y_1 = m(x - x_1)$$

Use the point-slope formula.

$$y - 4 = 2[x - (-2)]$$

Substitute the slope, 2, and the coordinates of the given point, (−2, 4), into the point-slope formula.

$$y - 4 = 2(x + 2)$$

Solve for *y*.

Example 1 – *Solution*

cont'd

$$y - 4 = 2x + 4$$

$$y = 2x + 8$$

The equation of the line is $y = 2x + 8$.

Find the equation of a line given two points

Find the equation of a line given two points

The point-slope formula and the formula for slope are used to find the equation of a line when two points are known.

Example 2

Find the equation of the line containing the points $P_1(2, 3)$ and $P_2(4, 1)$.

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 2}$$

Find the slope. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 1)$.

$$= \frac{-2}{2}$$

$$= -1$$

Example 2 – *Solution*

cont'd

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x - 2)$$

Substitute the slope and the coordinates of either one of the known points into the point-slope formula.

$$y - 3 = -x + 2$$

Solve for *y*.

$$y = -x + 5$$

The equation of the line is $y = -x + 5$.

Application problems

Application problems

Linear functions can be used to model a variety of applications in science and business. For each application, data are collected and the independent and dependent variables are selected. Then a linear function is that models the data is determined.

Example 3

In 2000, there were approximately 50,000 centenarians (people 100 years old or older). Data from the Census Bureau show that this population is expected to increase through the year 2020 at a rate of approximately 4250 centenarians per year. Find a linear function that approximates the population of centenarians in terms of the year. Use your function to approximate the number of centenarians in 2015.

Example 3

cont'd

Strategy:

Select the independent and dependent variables. Because we want to determine the population of centenarians, that quantity is the *dependent* variable, y. The year is the *independent* variable.

From the data, the ordered pair (2000, 50,000) gives the coordinates of a point on the line. The slope of the line is the *rate of increase*, 4250 centenarians per year.

Example 3 – *Solution*

$$y - y_1 = m(x - x_1)$$

Use the point-slope formula.

$$y - 50{,}000 = 4250(x - 2000)$$

$m = 4250$;
$(x_1, y_1) = (2000, 50{,}000)$

$$y - 50{,}000 = 4250x - 8{,}500{,}000$$

$$y = 4250x - 8{,}450{,}000$$

The linear function is $f(x) = 4250x - 8{,}450{,}000$.

Example 3 – *Solution*

cont'd

$$f(x) = 4250x - 8{,}450{,}000$$

$$f(2015) = 4250(2015) - 8{,}450{,}000$$

Evaluate the function at 2015 to predict the number of centenarians in 2015.

$$= 8{,}563{,}750 - 8{,}450{,}000$$

$$= 113{,}750$$

The function gives an estimate of 113,750 centenarians in 2015.