CHAPTER 3

Linear Functions and Inequalities in Two Variables

3.4 Slope of a Straight Line

Objectives

1 Find the slope of a line given two points

2 Graph a line given a point and the slope

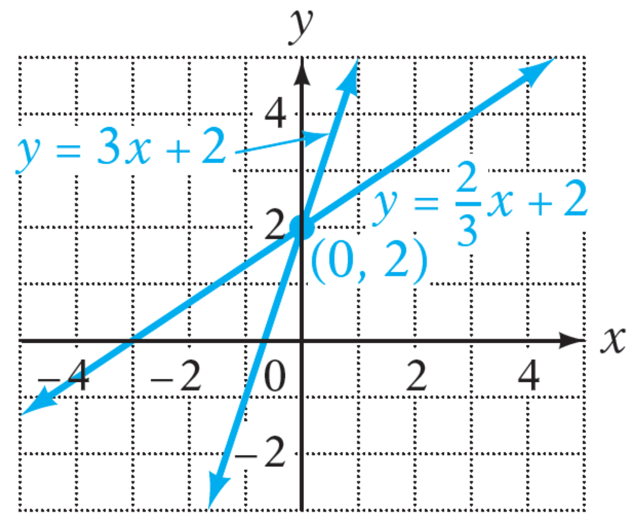
3 Average rate of change

Find the slope of a line given two points

Find the slope of a line given two points

The graphs of $y = 3x + 2$ and $y = \frac{2}{3}x + 2$ are shown at the left. Each graph crosses the *y*-axis at the point whose coordinates are (0, 2), but the graphs have different slants.

The **slope** of a line is a measure of the slant of the line.
The symbol for slope is *m.*
The slope of a line containing two points is the ratio of the change in the *y* values between the two points to the change in the *x* values.



Find the slope of a line given two points

Let us state a formula for slope.

SLOPE FORMULA

The slope of the line containing the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Frequently, the Greek letter Δ (delta) is used to designate the change in a variable.

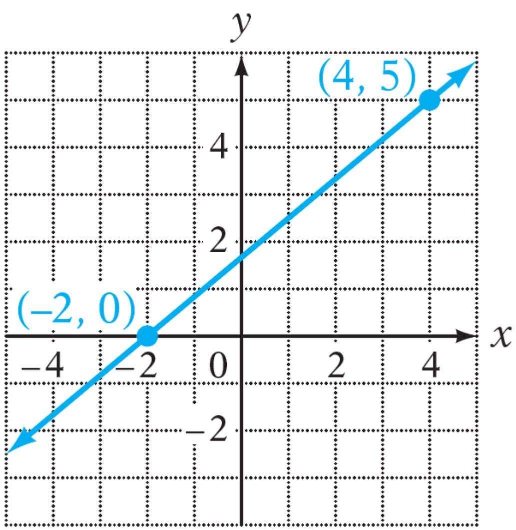
Find the slope of a line given two points

Using this notation, we can write the equations for the change in *y* and the change in *x* as follows:

Change in $y = y_2 - y_1 = \Delta y$ Change in $x = x_2 - x_1 = \Delta x$

With this notation, the slope formula is written $m = \frac{\Delta y}{\Delta x}$.

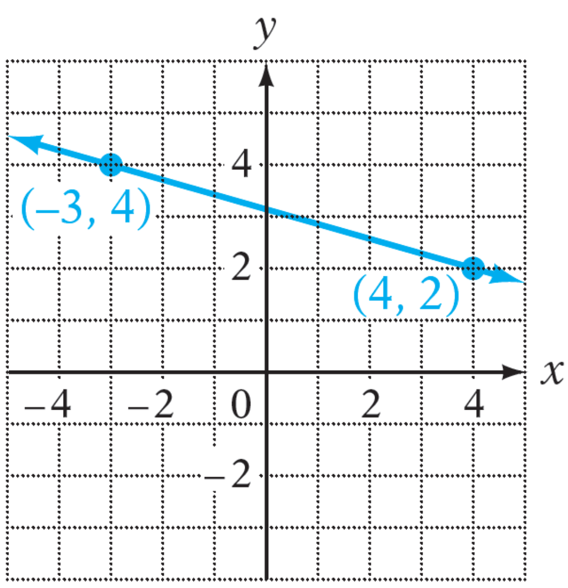
A line that slants upward to the right always has a **positive slope.**



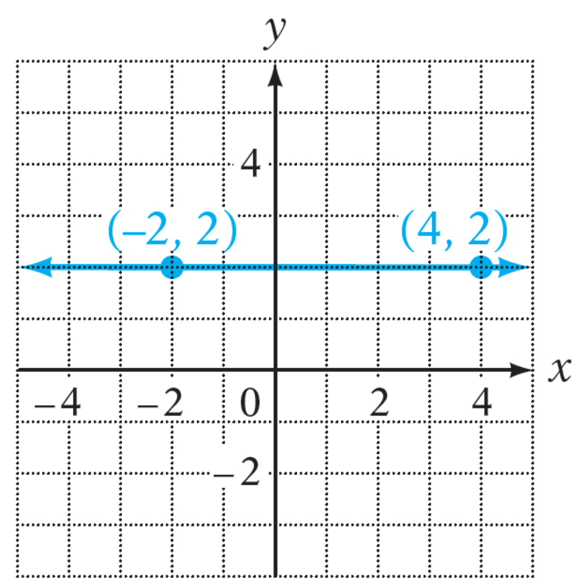
Positive slope

Find the slope of a line given two points

A line that slants downward to the right always has a **negative slope.** A horizontal line has **zero slope.**



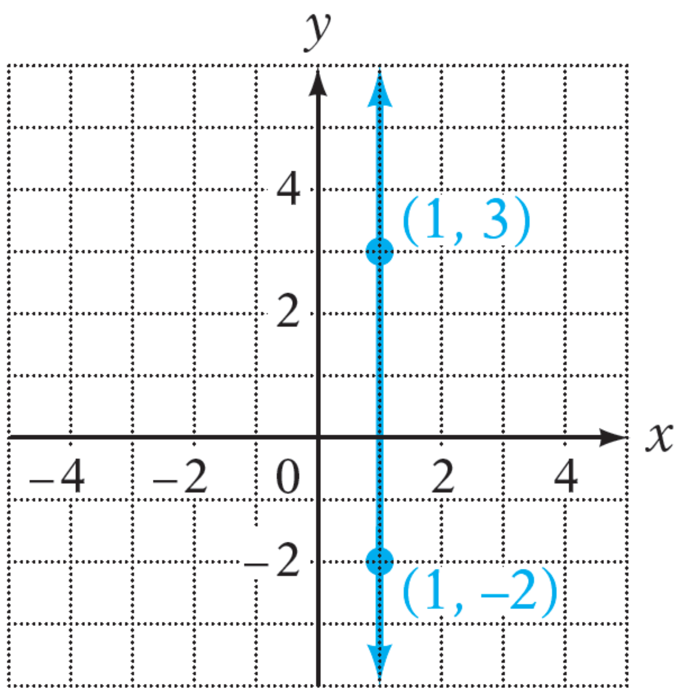
Negative slope



Zero slope

Find the slope of a line given two points

The slope of a vertical line is **undefined.**



Undefined slope

Example 1

Find the slope of the line containing the points whose coordinates are (2, –5) and (–4, 2).

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-5)}{-4 - 2}$$

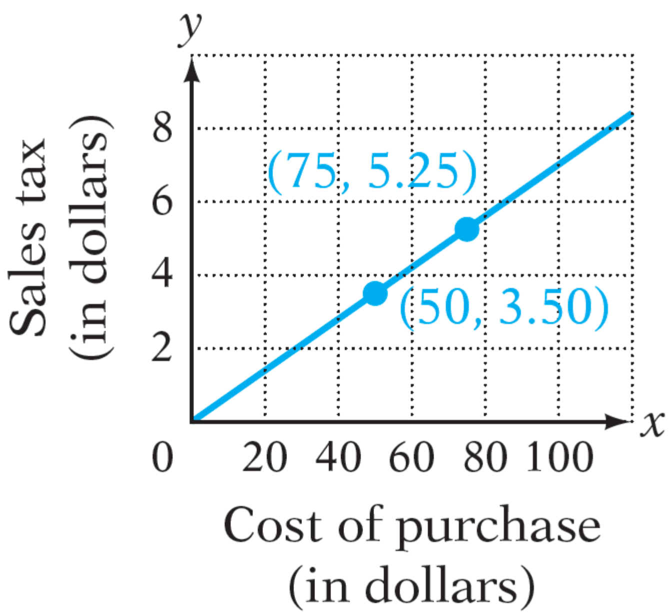
$$= \frac{7}{-6}$$

$$= -\frac{7}{6} \qquad \text{Let } P_1 = (2, -5) \text{ and } P_2 = (-4, 2).$$

The slope is $-\frac{7}{6}$.

Example 2

The graph shows the relationship between the cost of an item and the sales tax. Find the slope of the line between the two points shown on the graph. Write a sentence that states the meaning of the slope.



Solution:

$$m = \frac{5.25 - 3.50}{75 - 50} = \frac{1.75}{25} = 0.07$$

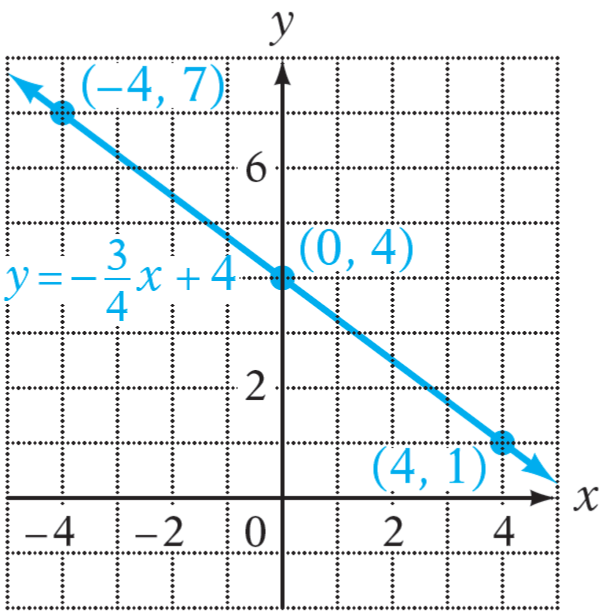
A slope of 0.07 means that the sales tax is $.07 per dollar.

Graph a line given a point and the slope

Graph a line given a point and the slope

The graph of the equation $y = -\frac{3}{4}x + 4$ is shown at the right. The points whose coordinates are (–4, 7) and (4, 1) are on the graph. The slope of the line is

$$m = \frac{7 - 1}{-4 - 4} = \frac{6}{-8} = -\frac{3}{4}$$



Note that the slope of the line has the same value as the coefficient of x.

Graph a line given a point and the slope

The y-intercept is found by replacing x by zero and then solving for y.

SLOPE-INTERCEPT FORM OF A STRAIGHT LINE

The equation $y = mx + b$ is called the **slope-intercept form** of a straight line. The slope of the line is m, the coefficient of x. The coordinates of the y-intercept are $(0, b)$.

When the equation of a straight line is in the form $y = mx + b$, the graph can be drawn by using the slope and the y-intercept. First locate the y-intercept. Use the slope to find a second point on the line. Then draw a line through the two points.

Example 3

Graph $y = -\frac{3}{2}x + 4$ by using the slope and the y-intercept.

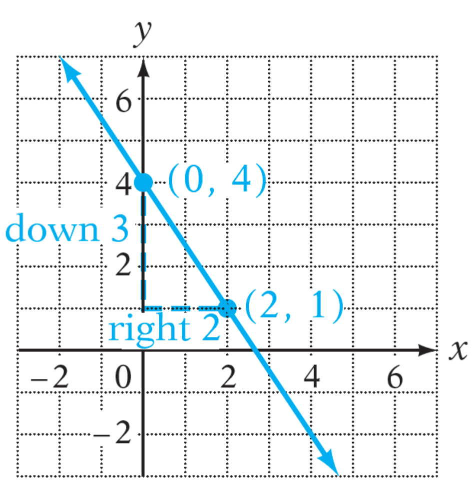
Solution:

y-intercept: $(0, 4)$

Determine the y-intercept from the constant term.

$$m = -\frac{3}{2} = \frac{-3}{2} = \frac{\text{change in } y}{\text{change in } x}$$

Move the negative sign into the numerator of the slope fraction.



Beginning at the y-intercept, whose coordinates are $(0, 4)$, move down 3 units and right 2 units. $(2, 1)$ are the coordinates of a second point on the graph.

Draw a line through the points whose coordinates are $(0, 4)$ and $(2, 1)$.

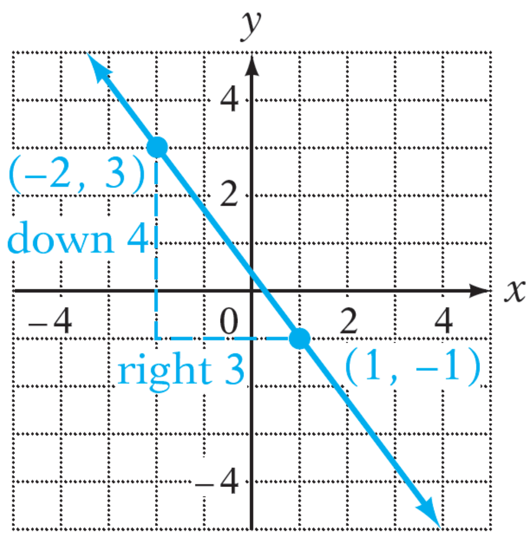
Example 4

Graph the line that passes through the point $P(–2, 3)$ and has slope $-\frac{4}{3}$.

Solution:

$$m = -\frac{4}{3} = \frac{-4}{3} = \frac{\text{change in } y}{\text{change in } x}$$

Move the negative sign into the numerator of the slope fraction.



Locate (–2, 3). Beginning at that point, move down 4 units and then right 3 units. (1, –1) are the coordinates of a second point on the line.

Draw a line through the points whose coordinates are (–2, 3) and (1, –1).

Average rate of change

Average rate of change

Slope measures the rate at which one quantity changes with respect to a second quantity. Straight lines have a constant slope. No matter which two points on the line are chosen, the slope of the line between the two points is the same.

If a graph is not a straight line, the slope of the line between two points on the graph may be different from the slope of the line between two other points.

In such cases, the **average rate of change** between any two points is the slope of the line between the two points.

Example 5

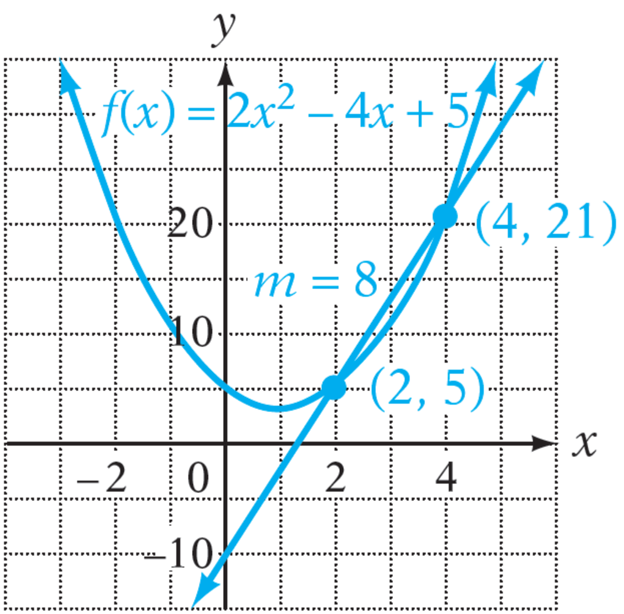
Find the average rate of change of $f(x) = 2x^2 - 4x + 5$ between the points whose x-coordinates are $x_1 = 2$ and $x_2 = 4$.

Solution:

Find the coordinates of each point by finding the y-coordinate for the given x-coordinate.

$$y_1 = f(x_1)$$

$$= 2(2)^2 - 4(2) + 5 = 5 \quad \mathbf{x_1 = 2}$$



The first point is $P_1(2, 5)$.

Example 5 – *Solution*

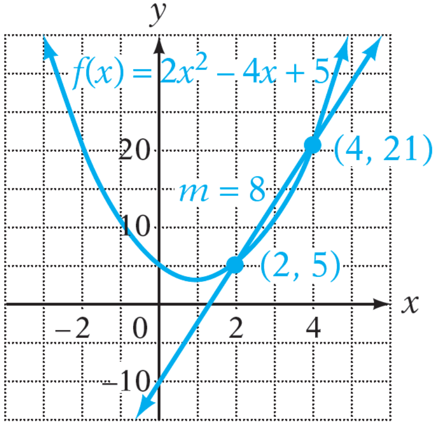
cont'd

$$y_2 = f(x_2)$$
$$= 2(4)^2 - 4(4) + 5 = 21 \qquad x_2 = 4$$

The second point is $P_2(4, 21)$.

To find the average rate of change between the two points, find the slope of the line between $P_1(2, 5)$ and $P_2(4, 21)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{21 - 5}{4 - 2} = \frac{16}{2} = 8$$



The average rate of change between the two points is 8.

Example 7

Find the average annual rate of change in the median salary of Boston Red Sox players between 1995 and 2010. Round to the nearest thousand dollars.

Solution:

In 1995, the median salary was 282,500: (1995, 282,500)

In 2010, the median salary was 3,750,000: (2010, 3,750,000)

$$m = \frac{3{,}750{,}000 - 282{,}500}{2010 - 1995} = \frac{3{,}467{,}500}{15} \approx 231{,}000$$

The average annual rate of change in median salary was approximately $231,000 per year.