CHAPTER 3

Linear Functions and Inequalities in Two Variables

3.1 The Rectangular Coordinate System

Objectives

1. Distance and midpoint formulas
2. Graph an equation in two variables

Distance and midpoint formulas

Distance and midpoint formulas

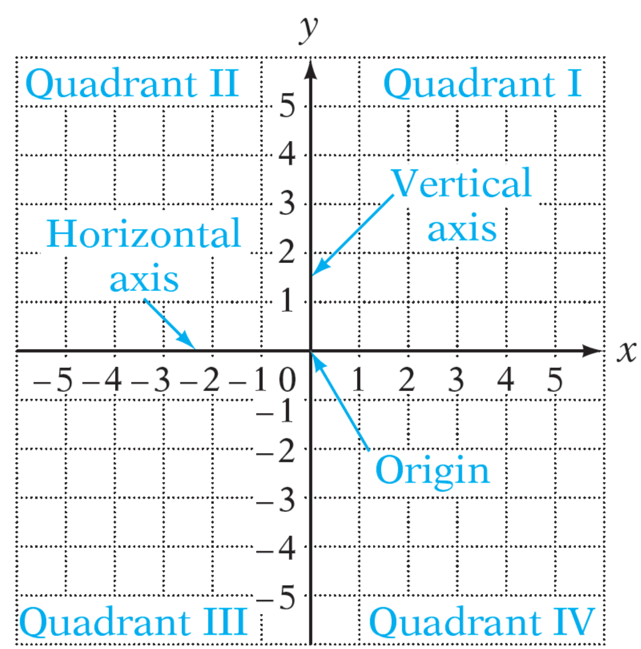
Before the 15th century, geometry and algebra were considered separate branches of mathematics. That changed when René Descartes, a French mathematician who lived from 596 to 1650, founded **analytic geometry**. In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin**. The two lines are called **coordinate axes**, or simply **axes.**

Distance and midpoint formulas

The axes determine a **plane**, which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants.** The quadrants are numbered counterclockwise from I to IV.

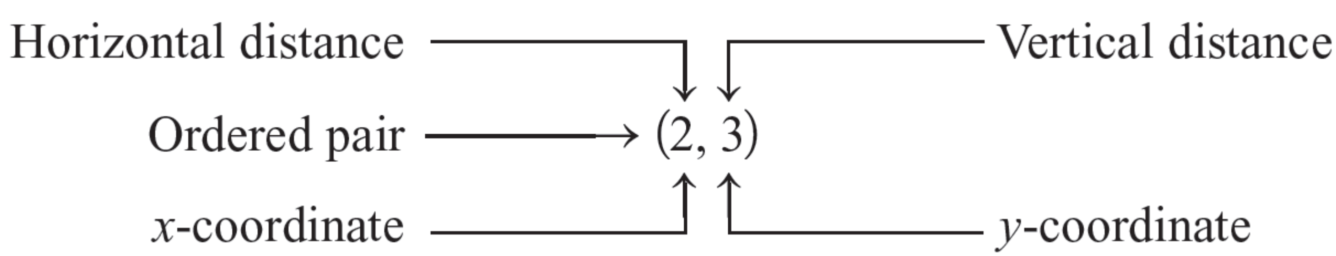
Each point in the plane can be identified by a pair of numbers called an **ordered pair**.
The first number of the pair measures a horizontal distance and is called the **abscissa**.



Distance and midpoint formulas

The second number of the pair measures a vertical distance and is called the **ordinate**. The **coordinates** of the point are the numbers in the ordered pair associated with the point.

The abscissa is also called the **first coordinate**, or ***x*-coordinate**, of the ordered air, and the ordinate is also called the **second coordinate**, or ***y*-coordinate**, of the ordered pair.



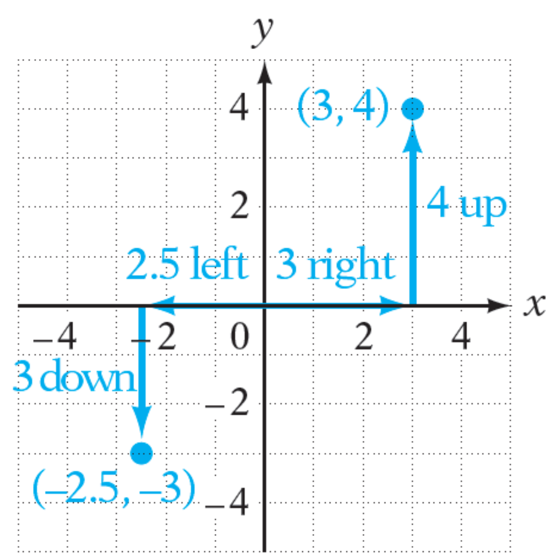
Distance and midpoint formulas

When drawing a rectangular coordinate system, we often label the horizontal axis *x* and the vertical axis *y*. In this case, the coordinate system is called an ***xy*-coordinate system**. To graph or plot a point in the *xy*-coordinate system, place a dot at the location given by the ordered pair.

The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the *xy*-coordinate system.

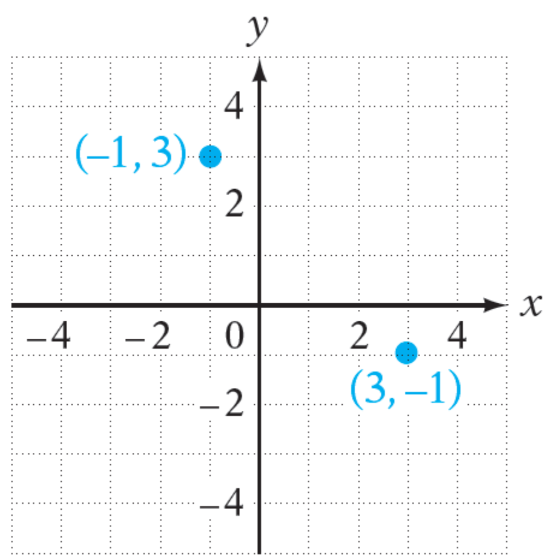
Distance and midpoint formulas

The points whose coordinates are (3, 4) and (–2.5, –3) are graphed in the figure.



Distance and midpoint formulas

The points whose coordinates are (3, –1) and (–1, 3) are graphed in the figure. Note that the graphs are in different locations. The *order* of the coordinates of an ordered pair is important.

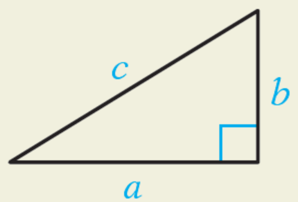


Distance and midpoint formulas

The distance between two points in an *xy*-coordinate system can be calculated by using the Pythagorean Theorem.

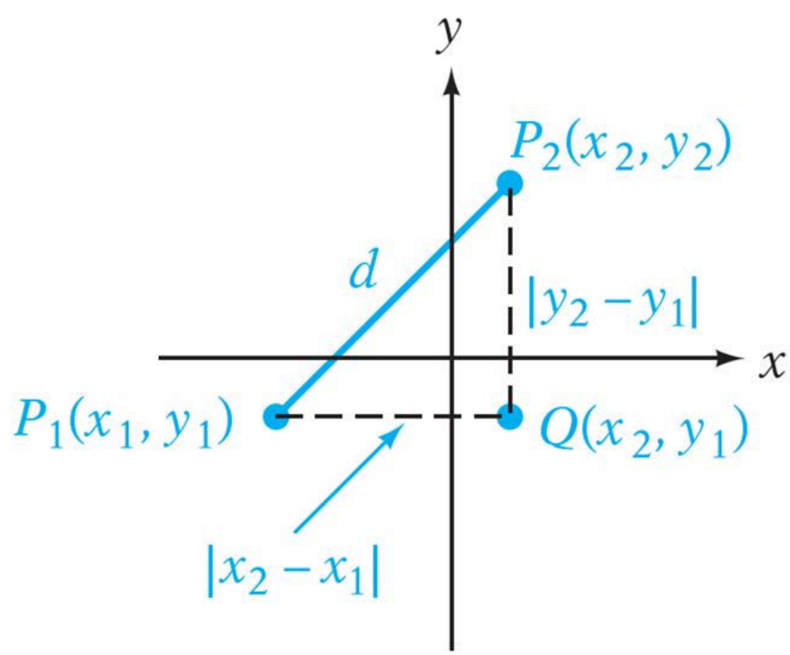
PYTHAGOREAN THEOREM

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.



Distance and midpoint formulas

Consider the two points and the right triangle shown below. The vertical distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $|y_2 - y_1|$. The horizontal distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $|x_2 - x_1|$.



Distance and midpoint formulas

THE DISTANCE FORMULA

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points in the plane, then the distance d between the two points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1

Find the exact distance between the points whose coordinates are (–3, 2) and (4, –1).

Solution:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use the distance formula.

$$= \sqrt{[4 - (-3)]^2 + (-1 - 2)^2}$$

$(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (4, -1)$.

Example 1 – *Solution*

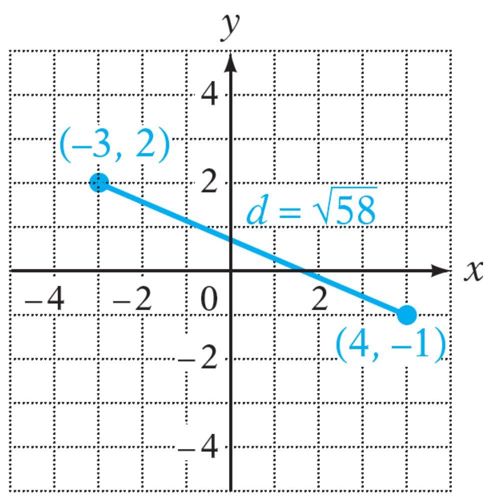
cont'd

$$= \sqrt{7^2 + (-3)^2} \qquad = \sqrt{49 + 9}$$

$$= \sqrt{58}$$

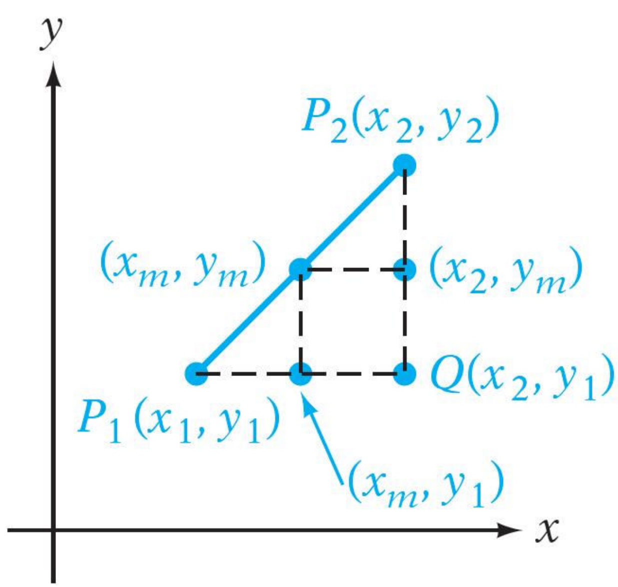
The distance between the points is $\sqrt{58}$.

See the following graph.



Distance and midpoint formulas

The **midpoint of a line segment** is equidistant from its endpoints. The coordinates of the midpoint of the line segment P_1P_2 are (x_m, y_m). The intersection of the horizontal line segment through P_1 and the vertical line segment through P_2 is Q, with coordinates (x_2, y_1).



Distance and midpoint formulas

THE MIDPOINT FORMULA

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are the endpoints of a line segment, then the coordinates (x_m, y_m) of the midpoint of the line segment are given by

$$x_m = \frac{x_1 + x_2}{2} \quad \text{and} \quad y_m = \frac{y_1 + y_2}{2}$$

Example 2

Find the coordinates of the midpoint of the line segment with endpoints $P_1(-5, 4)$ and $P_2(1, -3)$.

Solution:

$$x_m = \frac{x_1 + x_2}{2} \qquad y_m = \frac{y_1 + y_2}{2}$$

Use the midpoint formula.

$$= \frac{-5 + 1}{2} \qquad = \frac{4 + (-3)}{2}$$

$(x_1, y_1) = (-5, 4)$ and $(x_2, y_2) = (1, -3)$.

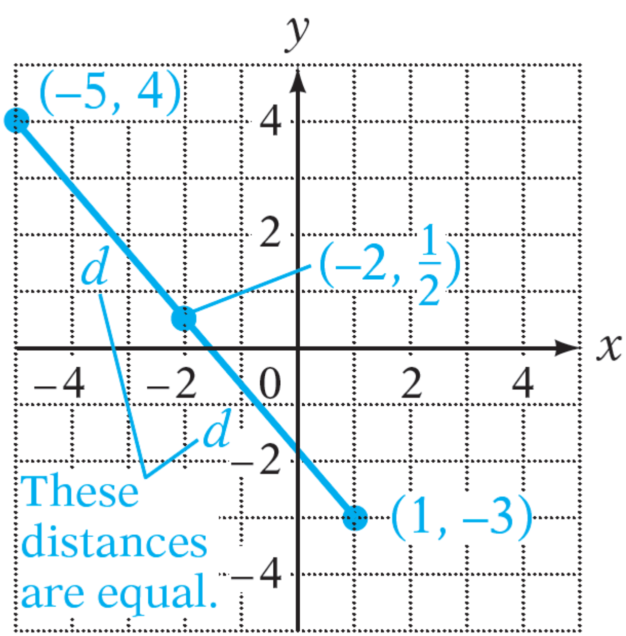
$$= -2 \qquad = \frac{1}{2}$$

The coordinates of the midpoint are $\left(-2, \frac{1}{2}\right)$.

Example 2 – *Solution*

cont'd

See the following graph.



Graph an equation in two variables

Graph an equation in two variables

The *xy*-coordinate system is used to graph equations in two variables. Examples of equations in two variables are shown below.

$$y = 3x + 7$$

$$y = x^2 - 4x + 3$$

$$x^2 + y^2 = 25$$

$$x = \frac{y}{y^2 + 4}$$

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

Graph an equation in two variables

In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

Example 3

Determine the ordered-pair solution of $y = \dfrac{x}{x - 2}$ that corresponds to $x = 4$.

Solution:

$$y = \frac{x}{x - 2}$$

$$y = \frac{4}{4 - 2}$$

Replace x by 4 and solve for y.

$$y = 2$$

The ordered-pair solution is (4, 2).

Graph an equation in two variables

The ordered-pair solutions of an equation in two variables can be graphed in a rectangular coordinate system.

Every ordered pair on the graph of an equation is a solution of the equation, and every ordered-pair solution of an equation gives the coordinates of a point on the graph of the equation.

Example 4

Graph $y = \frac{1}{2}x + 1$ by plotting the solutions of the equation when $x = -4, -2, 0, 2, \text{and } 4,$ and then connecting the points with a smooth graph.

Example 4 – *Solution*

Determine the ordered-pair solutions (x, y) for the given values of x. Plot the points and then connect the points with a smooth graph.

x	$y = \frac{1}{2}x + 1$	y	(x, y)
-4	$y = \frac{1}{2}(-4) + 1 = -1$	-1	$(-4, -1)$
-2	$y = \frac{1}{2}(-2) + 1 = 0$	0	$(-2, 0)$
0	$y = \frac{1}{2}(0) + 1 = 1$	1	$(0, 1)$
2	$y = \frac{1}{2}(2) + 1 = 2$	2	$(2, 2)$
4	$y = \frac{1}{2}(4) + 1 = 3$	3	$(4, 3)$