Solving Equations and Inequalities

CHAPTER 2

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2.6 Applications: Problems Involving Percent

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Objectives

1 Investment problems

2 Percent mixture problems

Investment problems

Investment problems

The annual simple interest that an investment earns is given by the equation $\boldsymbol{I = Pr}$, where I is the simple interest, P is the principal, or the amount invested, and r is the simple interest rate.

Investment problems

The equation *I* = *Pr* is used to solve the investment problem below.

Solve: An investor has a total of $10,000 to deposit in two simple interest accounts. On one account, the annual simple interest rate is 7%. On the second account, the annual simple interest rate is 8%. How much should be invested in each account so that the total annual interest earned is $785?

STRATEGY FOR SOLVING A PROBLEM INVOLVING MONEY DEPOSITED IN TWO SIMPLE INTEREST ACCOUNTS

- For each amount invested, use the equation $Pr = I$. Write a numerical or variable expression for the principal, the interest rate, and the interest earned. The results can be recorded in a table.

Investment problems

The sum of the amounts invested is $10,000.

Amount invested at 7%: x

Amount invested at 8%: $10,000 – x

	Principal, P	$\cdot$	Interest rate, r	$=$	Interest earned, I
Amount at 7%	x	$\cdot$	0.07	$=$	$0.07x$
Amount at 8%	$10,000 - x$	$\cdot$	0.08	$=$	$0.08(10,000 - x)$

► Determine how the amounts of interest earned on the individual investments are related. For example, the total interest earned by both accounts may be known, or it may be known that the interest earned on one account is equal to the interest earned on the other account.

Investment problems

The sum of the interest earned by the two investments equals the total annual interest earned ($785).

$$0.07x + 0.08(10{,}000 - x) = 785$$

The interest earned on the 7% account plus the interest earned on the 8% account equals the total annual interest earned.

$$0.07x + 800 - 0.08x = 785$$

$$-0.01x + 800 = 785$$

$$-0.01x = -15$$

$$x = 1500$$

The amount invested at 7% is $1500.

Investment problems

$10{,}000 - x = 10{,}000 - 1500 = 8500$ Substitute the value of *x* into the variable expression for the amount invested at 8%.

The amount invested at 7% is $1500.

The amount invested at 8% is $8500.

Example 1

An investment counselor invested 75% of a client's money into a 9% annual simple interest money market fund. The remainder was invested in 6% annual simple interest government securities. Find the amount invested in each account if the total annual interest earned is $3300.

Strategy:

- Amount invested: x

  Amount invested at 9%: $0.75x$

  Amount invested at 6%: $0.25x$

Example 1

cont'd

	Principal	$\cdot$	Rate	$=$	Interest
Amount at 9%	$0.75x$	$\cdot$	0.09	$=$	$0.09(0.75x)$
Amount at 6%	$0.25x$	$\cdot$	0.06	$=$	$0.06(0.25x)$

- The sum of the interest earned by the two investments equals the total annual interest earned ($3300).

Example 1 – *Solution*

$0.09(0.75x) + 0.06(0.25x) = 3300$

$0.0675x + 0.015x = 3300$

$0.0825x = 3300$

The interest earned on the 9% account plus the interest earned on the 6% account equals the total annual interest earned.

$x = 40{,}000$

The amount invested is $40,000.

$0.75x = 0.75(40{,}000) = 30{,}000$

Find the amount invested at 9%.

$0.25x = 0.25(40{,}000) = 10{,}000$

Find the amount invested at 6%.

The amount invested at 9% is $30,000.

The amount invested at 6% is $10,000.

Percent mixture problems

Percent mixture problems

The amount of a substance in a solution can be given as a percent of the total solution.

For example, in a 5% saltwater solution, 5% of the total solution is salt. The remaining 95% is water.

The solution of a percent mixture problem is based on the equation $\boldsymbol{Q = Ar}$, where Q is the quantity of a substance in the solution, r is the percent of concentration, and A is the amount of solution.

Example 2

A chemist wishes to make 3 L of a 7% acid solution by mixing a 9% acid solution and a 4% acid solution. How many liters of each solution should the chemist use?

Strategy:

Liters of 9% solution: x
Liters of 4% solution: $3 – x$

	Amount	Percent	Quantity
9%	x	0.09	$0.09x$
4%	$3 - x$	0.04	$0.04(3 - x)$
7%	3	0.07	$0.07(3)$

The sum of the quantities before mixing is equal to the quantity after mixing.

Example 2 – *Solution*

$$0.09x + 0.04(3 - x) = 0.07(3)$$

The amount of acid in the 9% solution plus the amount of acid in the 4% solution equals the amount of acid in the 7% solution.

$$0.09x + 0.12 - 0.04x = 0.21$$

$$0.05x + 0.12 = 0.21$$

$$0.05x = 0.09$$

$$x = 1.8$$

1.8 L of the 9% solution are needed.

$$3 - x = 3 - 1.8 = 1.2$$

Find the amount of the 4% solution needed.

The chemist needs 1.8 L of the 9% solution and 1.2 L of the 4% solution.