CHAPTER 2

Solving Equations and Inequalities

2.4 Geometry Problems

Objectives

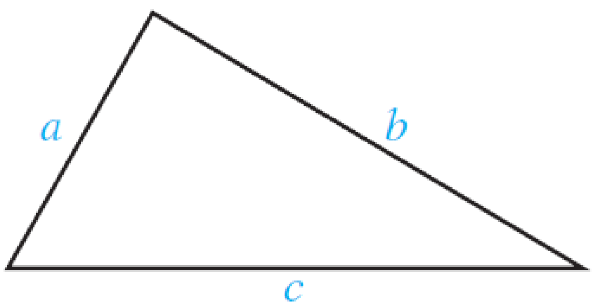
1. Perimeter problems
2. Problems involving angles formed by intersecting lines
3. Problems involving the angles of a triangle

Perimeter problems

Perimeter problems

The **perimeter** of a plane geometric figure is a measure of the distance around the figure. Perimeter is used, for example, in buying fencing for a lawn and in determining how much baseboard is needed for a room.

The perimeter of a triangle is the sum of the lengths of the three sides. Therefore, if *a*, *b*, and *c* represent the lengths of the sides of *a* triangle, the perimeter *P* of the triangle is given by $\boldsymbol{P = a + b + c}$.

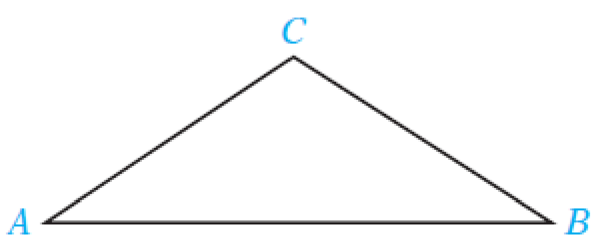


$P = a + b + c$

Perimeter problems

Two special types of triangles are shown below. An **isosceles triangle** has two sides of equal length. The two angles opposite the two sides of equal length are of equal measure.

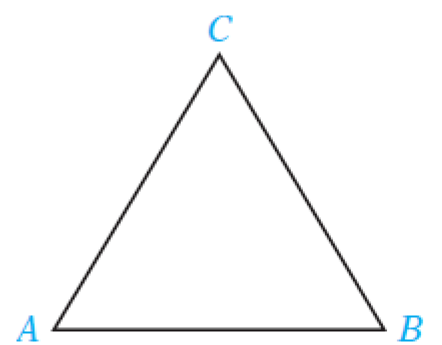
The three sides of an **equilateral triangle** are of equal length, and all three angles have the same measure.



Isosceles triangle

$AC = BC$

$\angle A = \angle B$



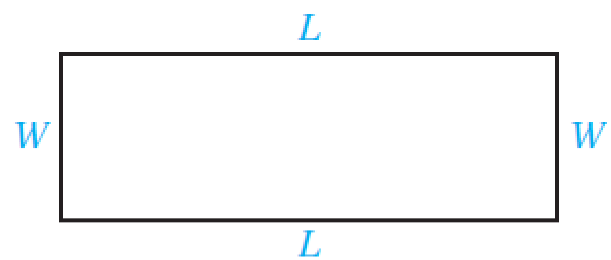
Equilateral triangle

$AB = AC = BC$

$\angle A = \angle B = \angle C$

Perimeter problems

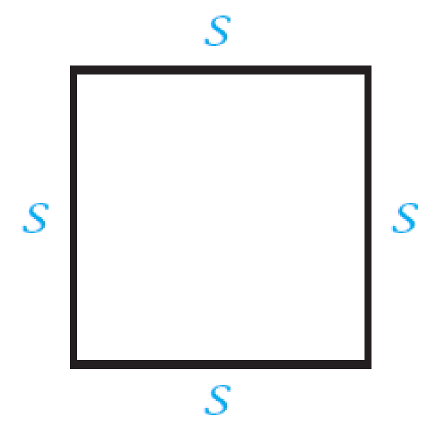
The perimeter of a rectangle is the sum of the lengths of the four sides. Let L represent the length and W represent the width of a rectangle. Then the perimeter P of the rectangle is given by $P = L + W + L + W$. Combine like terms and the formula is $\boldsymbol{P = 2L + 2W}$.



$P = 2L + 2W$

Perimeter problems

A square is a rectangle in which each side has the same length. Let *s* represent the length of each side of a square.
Then the perimeter *P* of the square is given by $P = s + s + s + s$.



$P = 4s$

Combine like terms and the formula is $\boldsymbol{P = 4s}$.

FORMULAS FOR PERIMETERS OF GEOMETRIC FIGURES

Perimeter of a triangle:	$P = a + b + c$
Perimeter of a rectangle:	$P = 2L + 2W$
Perimeter of a square:	$P = 4s$

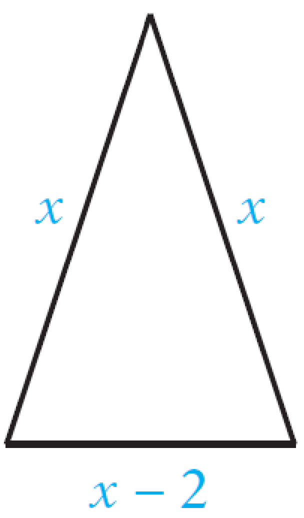
Example 1

The perimeter of an isosceles triangle is 25 ft. The length of the third side is 2 ft less than the length of one of the equal sides. Find the measures of the three sides of the triangle.

Strategy:

- Each equal side: x

  The third side: $x - 2$
- Use the equation for the perimeter of a triangle.



Example 1 – *Solution*

$P = a + b + c$	Use the formula for the perimeter of a triangle.
$25 = x + x + (x - 2)$	Substitute 25 for *P*. Substitute the variable expressions for the three sides of the triangle.
$25 = 3x - 2$	Solve the equation for *x*.
$27 = 3x$	
$9 = x$	

Example 1 – *Solution*

cont'd

$x - 2 = 9 - 2$ Substitute the value of x into the variable expression for the length of the third side.

$= 7$

Each of the equal sides measures 9 ft.

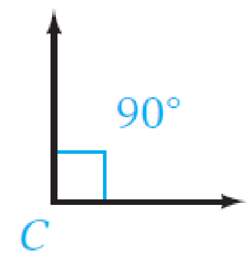
The third side measures 7 ft.

Problems involving angles formed by intersecting lines

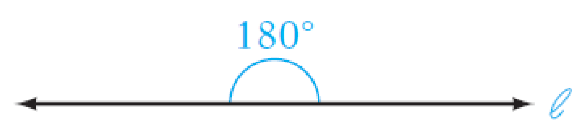
Problems involving angles formed by intersecting lines

A unit used to measure angles is **degree.** The symbol for degree is °. ∠ is the symbol for angle.
One complete revolution is 360°.

A 90° angle is called a **right angle.** The symbol ⊾ represents a right angle. Angle *C*(∠*C*) is a right angle.

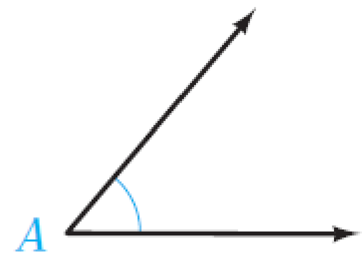


A 180° angle is called a **straight angle.** The angle at the right is a straight angle.

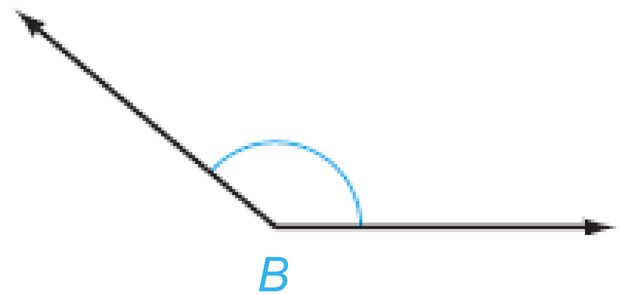


Problems involving angles formed by intersecting lines

An **acute angle** is an angle whose measure is between 0° and 90°. $\angle A$ at the right is an acute angle.

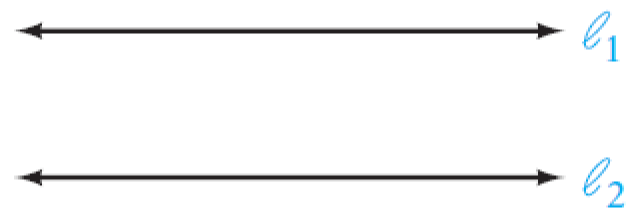


An **obtuse angle** is an angle whose measure is between 90° and 180°. $\angle B$ at the right is an obtuse angle.

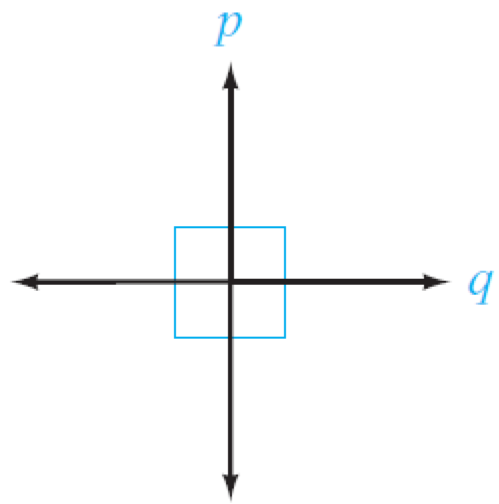


Problems involving angles formed by intersecting lines

Parallel lines never meet. The distance between them is always the same. The symbol $\parallel$ means "is parallel to." In the figure at the right, $\ell_1 \parallel \ell_2$.

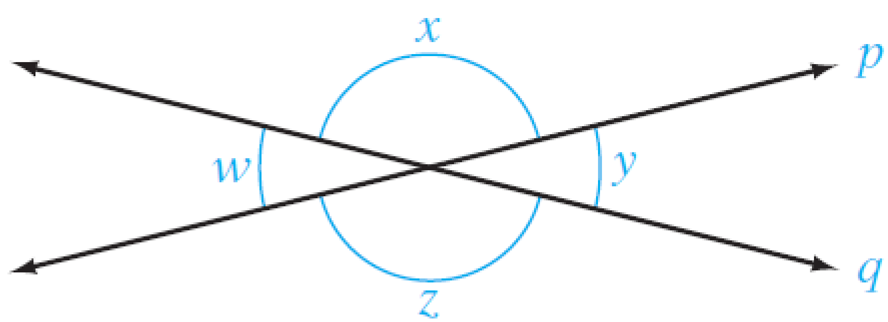


Perpendicular lines are intersecting lines that form right angles. The symbol $\perp$ means "is perpendicular to." In the figure at the right, $p \perp q$.



Problems involving angles formed by intersecting lines

Two angles that are on opposite sides of the intersection of two lines are called **vertical angles**. Vertical angles have the same measure. $\angle w$ and $\angle y$ are vertical angles. $\angle x$ and $\angle z$ are vertical angles.

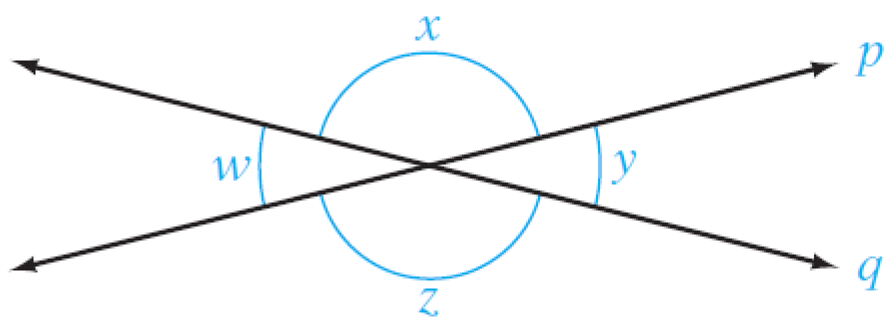


Vertical angles have the same measure.

$$\angle w = \angle y$$
$$\angle x = \angle z$$

Problems involving angles formed by intersecting lines

Two angles that share a common side are called **adjacent angles**. For the figure shown below, $\angle x$ and $\angle y$ are adjacent angles, as are $\angle y$ and $\angle z$, $\angle z$ and $\angle w$, and $\angle w$ and $\angle x$.



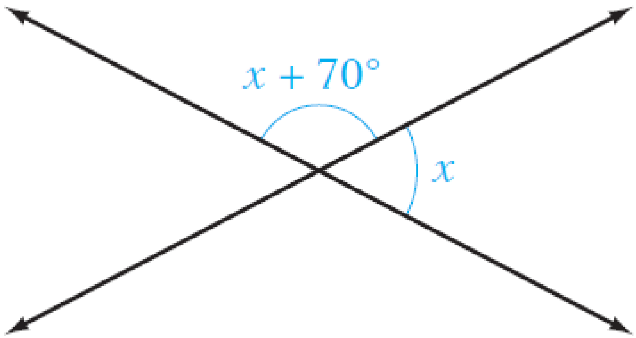
Problems involving angles formed by intersecting lines

Adjacent angles of intersecting lines are supplementary angles.

$$\angle x + \angle y = 180^\circ$$
$$\angle y + \angle z = 180^\circ$$
$$\angle z + \angle w = 180^\circ$$
$$\angle w + \angle x = 180^\circ$$

Example 2

Find *x.*



Strategy:

The angles labeled are adjacent angles of intersecting lines and are therefore supplementary angles. To find *x*, write an equation and solve for *x*.

Example 2 – *Solution*

$$x + (x + 70°) = 180°$$

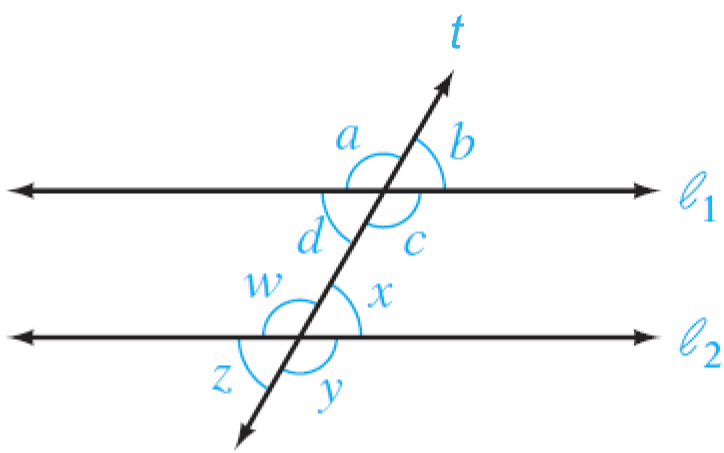
$$2x + 70° = 180°$$

$$2x = 110°$$

$$x = 55°$$

Problems involving angles formed by intersecting lines

A line that intersects two other lines at different points is called a **transversal.** If the lines cut by a transversal *t* are parallel lines and the transversal is not perpendicular to the parallel lines, all four acute angles have the same measure and all four obtuse angles have the same measure. In the figure below,

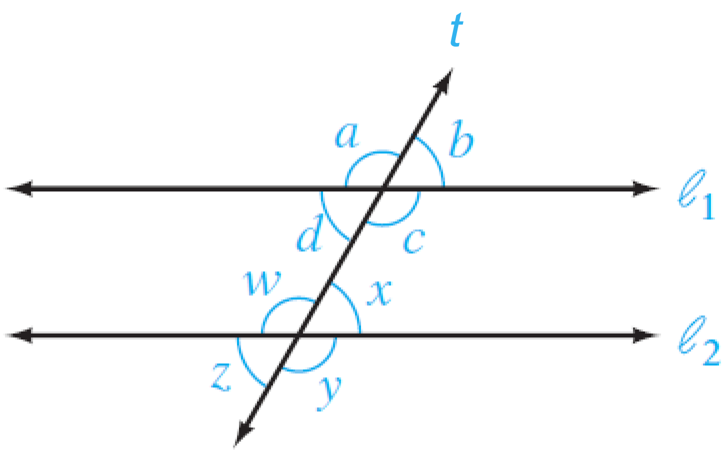


$$\angle b = \angle d = \angle x = \angle z$$
$$\angle a = \angle c = \angle w = \angle y$$

Problems involving angles formed by intersecting lines

Alternate interior angles are two nonadjacent angles that are on opposite sides of the transversal and between the lines. In the figure below, $\angle c$ and $\angle w$ are alternate interior angles, and $\angle d$ and $\angle x$ are alternate interior angles.

Alternate interior angles have the same measure.

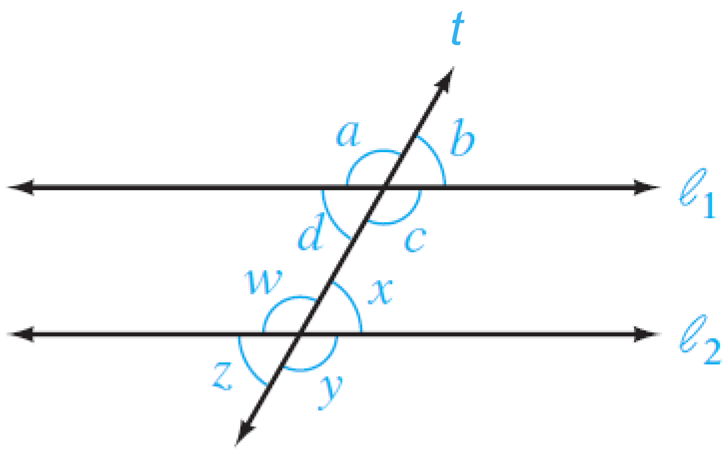


$$\angle c = \angle w$$
$$\angle d = \angle x$$

Problems involving angles formed by intersecting lines

Alternate exterior angles are two nonadjacent angles that are on opposite sides of the transversal and outside the parallel lines. In the figure below $\angle a$ and $\angle y$ are alternate exterior angles, and $\angle b$ and $\angle z$ are alternate exterior angles.

Alternate exterior angles have the same measure.



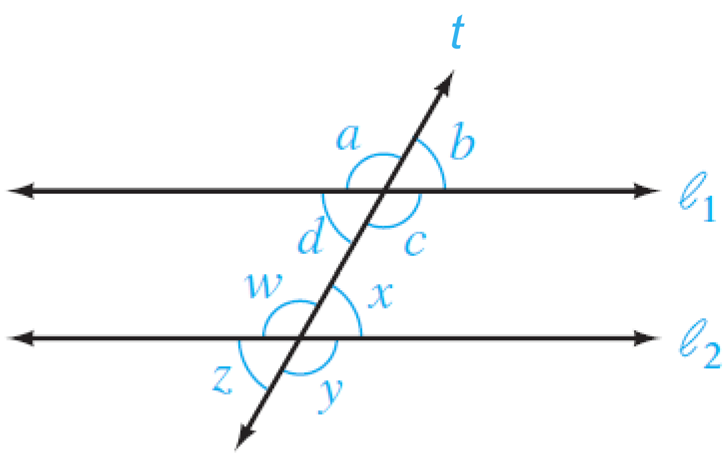
$$\angle a = \angle y$$
$$\angle b = \angle z$$

Problems involving angles formed by intersecting lines

Corresponding angles are two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. In the figure below, the following pairs of angles are corresponding angles:

$\angle a$ and $\angle w$, $\angle d$ and $\angle z$, $\angle b$ and $\angle x$, and $\angle c$ and $\angle y$.

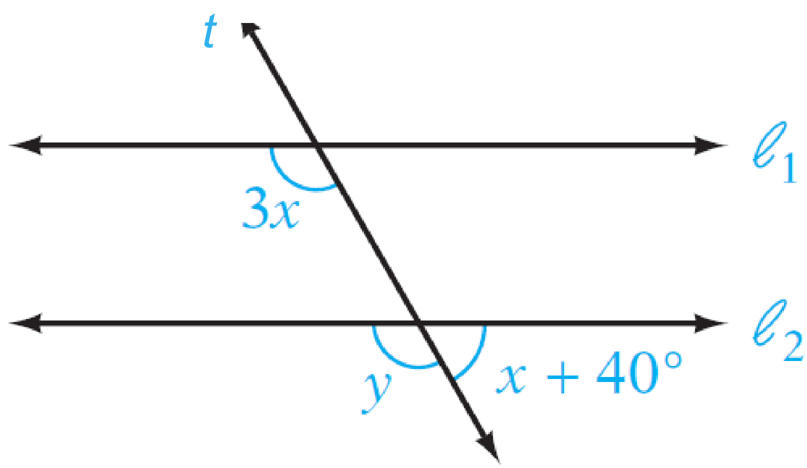
Corresponding angles have the same measure.



$$\angle a = \angle w$$
$$\angle d = \angle z$$
$$\angle b = \angle x$$
$$\angle c = \angle y$$

Example 3

Given $\ell_1 \parallel \ell_2$, find x.



Strategy:

$3x = y$ because corresponding angles have the same measure. $y + (x + 40°) = 180°$ because adjacent angles of intersecting lines are supplementary angles.

Substitute $3x$ for y and solve for x.

Example 3 – *Solution*

$$y + (x + 40°) = 180°$$

$$3x + (x + 40°) = 180°$$

$$4x + 40° = 180°$$

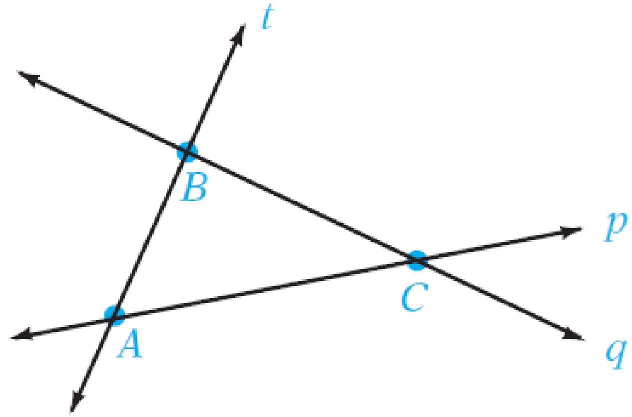
$$4x = 140°$$

$$x = 35°$$

Problems involving the angles of a triangle

Problems involving the angles of a triangle

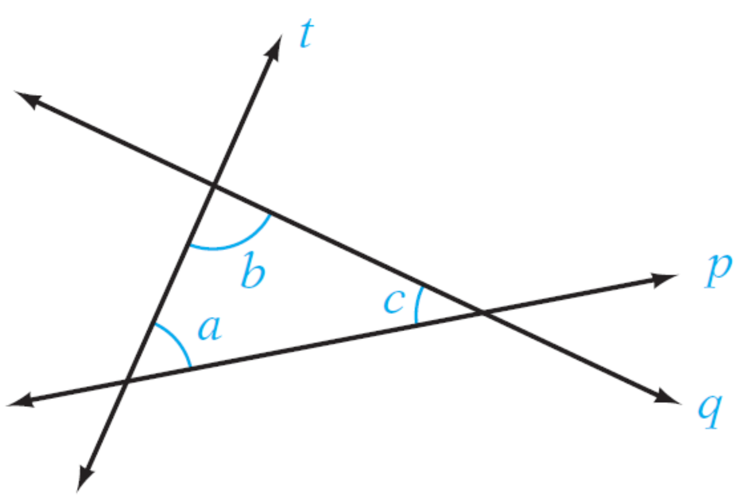
If the lines cut by a transversal are not parallel lines, the three lines will intersect at three points. In the following figure, the transversal *t* intersects lines *p* and *q*. The three lines intersect at points *A*, *B*, and *C*. The geometric figure formed by line segments *AB*, *BC*, and *AC* is a **triangle.**



Problems involving the angles of a triangle

The angles within the region enclosed by the triangle are called **interior angles**. In the following figure, angles *a*, *b*, and *c* are interior angles. The sum of the measures of the interior angles is 180°.

$$\angle a + \angle b + \angle c = 180°$$

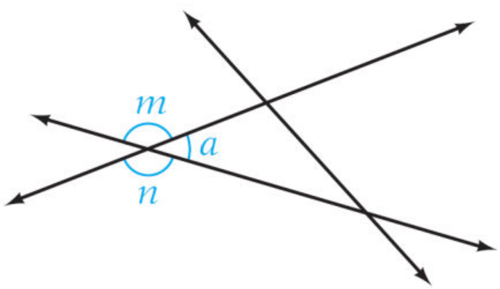


Problems involving the angles of a triangle

THE SUM OF THE MEASURES OF THE INTERIOR ANGLES OF A TRIANGLE

The sum of the measures of the interior angles of a triangle is 180°.

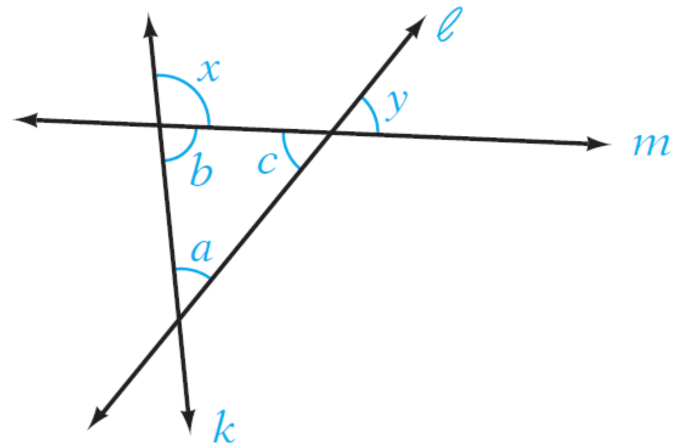
An angle adjacent to an interior angle is an **exterior angle.** In the figure below, angles *m* and *n* are exterior angles for angle *a*. The sum of the measures of an interior and an exterior angle is 180°.



$$\angle a + \angle m = 180°$$
$$\angle a + \angle n = 180°$$

Example 4

Given that $\angle a = 45°$ and $\angle x = 100°$, find the measures of angles *b*, *c*, and *y*.



Strategy:

- To find the measure of $\angle b$, use the fact that $\angle b$ and $\angle x$ are supplementary angles.

- To find the measure of $\angle c$, use the fact that the sum of the measures of the interior angles of a triangle is 180°.

Example 4

cont'd

- To find the measure of $\angle y$, use the fact that $\angle c$ and $\angle y$ are vertical angles.

Solution:

$$\angle b + \angle x = 180°$$

$\angle b$ and $\angle x$ are supplementary angles.

$$\angle b + 100° = 180°$$

$$\angle b = 80°$$

Example 4 – *Solution*

cont'd

$$\angle a + \angle b + \angle c = 180°$$

The sum of the measures of the interior angles of a triangle is 180°

$$45° + 80° + \angle c = 180°$$

$$125° + \angle c = 180°$$

$$\angle c = 55°$$

$$\angle y = \angle c = 55°$$

$\angle$ *c* and $\angle$ *y* are vertical angles.