CHAPTER

1

Real Numbers and Variable Expressions

1.3 Rational Numbers

Objectives

1. Write rational numbers as decimals
2. Add and subtract rational numbers
3. Multiply and divide rational numbers
4. Convert among percents, fractions, and decimals

Write rational numbers as decimals

Write rational numbers as decimals

A **rational number** is the quotient of two integers. Therefore, a rational number is a number that can be written in the form $\frac{a}{b}$, where *a* and *b* are integers, and *b* is not zero. A rational number written in this way is commonly called a **fraction.**

Because an integer can be written as the quotient of the integer and 1, every integer is a rational number.

A number written in **decimal notation** is also a rational number.

A rational number written as a fraction can be written in decimal notation.

Example 1

Write $\frac{5}{8}$ as a decimal.

Solution:

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
-48 \\
\hline
20 \\
-16 \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

←This is called a **terminating decimal**.

← The remainder is zero.

$\frac{5}{8}$ = 0.625

Example 2

Write $\frac{4}{11}$ as a decimal.

Solution:

$$
\begin{array}{r}
0.3636\ldots \\
11\overline{)4.0000} \\
-3\ 3 \\
\hline
70 \\
-66 \\
\hline
40 \\
-33 \\
\hline
70 \\
-66 \\
\hline
4
\end{array}
$$

←This is called a **repeating decimal**.

← The remainder is never zero.

$\frac{4}{11} = 0.\overline{36}$ ←The bar over the digits 3 and 6 is used to show that these digits repeat.

Write rational numbers as decimals

Numbers that cannot be written as either a repeating decimal or a terminating decimal are called **irrational numbers**. For example, 2.45445444544445... is an irrational number.

Two other examples are $\sqrt{2}$ and π.

$$\sqrt{2} = 1.414213562... \qquad \pi = 3.141592654...$$

The three dots mean that the digits continue on and on without ever repeating or terminating. Although we cannot write a decimal that is exactly equal to $\sqrt{2}$ or to π, we can give approximations of these numbers.

Write rational numbers as decimals

The symbol $\approx$ is read "is approximately equal to."

Shown below are $\sqrt{2}$ rounded to the nearest thousandth and π rounded to the nearest hundredth.

$$\sqrt{2} \approx 1.414 \qquad \pi \approx 3.14$$

The rational numbers and the irrational numbers taken together are called the **real numbers**.

Add and subtract rational numbers

Add and subtract rational numbers

The sign rules for adding integers apply to addition of rational numbers.

To add or subtract rational numbers written as fractions, first rewrite the fractions as equivalent fractions with a common denominator.

A common denominator is the **least common multiple (LCM)** of the denominators.

The LCM of the denominators is also called the **lowest common denominator (LCD)**.

Example 3

Add: $-\frac{5}{6} + \frac{3}{10}$

Solution:

Prime factorizations of 6 and 10:

Find the LCM of the denominators 6 and 10.

$6 = 2 \cdot 3 \qquad 10 = 2 \cdot 5$

$\text{LCM} = 2 \cdot 3 \cdot 5 = 30$

$$-\frac{5}{6} + \frac{3}{10} = -\frac{25}{30} + \frac{9}{30}$$

Rewrite the fractions as equivalent fractions, using the LCM of the denominators as the common denominator.

Example 3 – *Solution*

cont'd

$$= \frac{-25 + 9}{30}$$

Add the numerators, and place the sum over the common denominator.

$$= \frac{-16}{30}$$

$$= -\frac{8}{15}$$

Write the answer in simplest form.

Add and subtract rational numbers

To add or subtract decimals, write the numbers so that the decimal points are in a vertical line. Then proceed as in the addition or subtraction of integers.

Write the decimal point in the answer directly below the decimal points in the problem.

Example 5

Add: 14.02 + 137.6 + 9.852

Solution:

$$\begin{array}{r} 14.02 \\ 137.6 \\ +\quad 9.852 \\ \hline 161.472 \end{array}$$

Write the decimals so that the decimal points are in a vertical line.

Write the decimal point in the sum directly below the decimal points in the problem.

Example 6

Add: –114.039 + 84.76

Solution:

$$\begin{array}{r} 114.039 \\ -\ 84.76 \\ \hline 29.279 \end{array}$$

The signs are different. Subtract the absolute value of the number with the smaller absolute value from the absolute value of the number with the larger absolute value.

–114.039 + 84.76

= –29.279

Attach the sign of the number with the larger absolute value.

Multiply and divide rational numbers

Multiply and divide rational numbers

The sign rules for multiplying and dividing integers apply to multiplication and division of rational numbers.

The product of two fractions is the product of the numerators divided by the product of the denominators.

A fraction is in **simplest form** when the numerator and denominator have no common factors other than 1.

Example 7

Multiply: $\frac{3}{8} \cdot \frac{12}{17}$

Solution:

$$\frac{3}{8} \cdot \frac{12}{17} = \frac{3 \cdot 12}{8 \cdot 17}$$

Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 17}$$

Write the prime factorization of each factor. Divide by the common factors.

$$= \frac{9}{34}$$

Multiply the numbers remaining in the numerator. Multiply the numbers remaining in the denominator.

Multiply and divide rational numbers

The **reciprocal** of a fraction is the fraction with the numerator and denominator interchanged.

For instance, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of $-\frac{5}{4}$ is $-\frac{4}{5}$. To divide fractions, multiply the dividend by the reciprocal of the divisor.

Example 8

Divide: $\frac{3}{10} \div \left(-\frac{18}{25}\right)$

Solution:

$$\frac{3}{10} \div \left(-\frac{18}{25}\right) = -\left(\frac{3}{10} \div \frac{18}{25}\right)$$

The signs are different. The quotient is negative.

$$= -\left(\frac{3}{10} \cdot \frac{25}{18}\right)$$

Change division to multiplication and invert the divisor.

$$= -\left(\frac{3 \cdot 25}{10 \cdot 18}\right)$$

Multiply the numerators.
Multiply the denominators.

$$= -\left(\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{2 \cdot \underset{1}{\cancel{5}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3}\right)$$

$$= -\frac{5}{12}$$

Multiply and divide rational numbers

To multiply decimals, multiply as in the multiplication of whole numbers.

Write the decimal point in the product so that the number of decimal places in the product equals the sum of the decimal places in the factors.

Example 9

Multiply: (–6.89)(0.00035)

Solution:

$$
\begin{array}{rl}
6.89 & 2 \text{ decimal places} \\
\times\ 0.00035 & 5 \text{ decimal places} \\
\hline
3445 & \\
2067 & \\
\hline
0.0024115 & 7 \text{ decimal places}
\end{array}
$$

Multiply the absolute values.

(–6.89)(0.00035) = –0.0024115

The signs are different. The product is negative.

Multiply and divide rational numbers

To divide decimals, move the decimal point in the divisor to make it a whole number. Move the decimal point in the dividend the same number of places to the right.

Place the decimal point in the quotient directly over the decimal point in the dividend. Then divide as in the division of whole numbers.

Example 10

Divide: $-0.394 \div 1.7$. Round to the nearest hundredth.

Solution:

$$1.7.\overline{)0.3.940}$$

Move the decimal point one place to the right in the divisor and in the dividend. Place the decimal point in the quotient.

$$\begin{array}{r} 0.231 \approx 0.23 \\ 17.\overline{)03.940} \\ -34 \\ \hline 54 \\ -51 \\ \hline 30 \\ -17 \\ \hline 13 \end{array}$$

The symbol ≈ is used to indicate that the quotient is an approximate value that has been rounded off.

$-0.394 \div 1.7 \approx -0.23$

The signs are different. The quotient is negative.

Convert among percents, fractions, and decimals

Convert among percents, fractions, and decimals

"A population growth rate of 3%," "a manufacturer's discount of 25%," and "an 8% increase in pay" are typical examples of the many ways in which percent is used in applied problems. **Percent** means "parts of 100." Thus 27% means 27 parts of 100.

In applied problems involving a percent, it is usually necessary either to rewrite the percent as a fraction or a decimal, or to rewrite a fraction or a decimal as a percent.

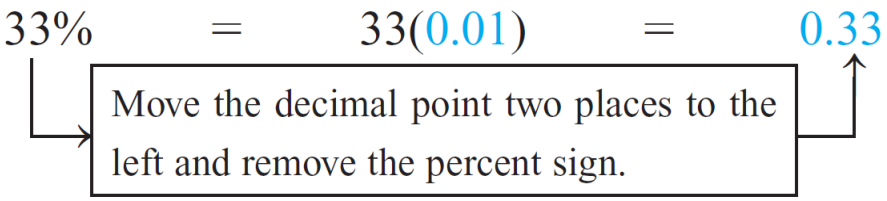
To write 27% as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

$$27\% = 27\left(\frac{1}{100}\right) = \frac{27}{100}$$

Convert among percents, fractions, and decimals

To write a percent as a decimal, remove the percent sign and multiply by 0.01.

To write 33% as a decimal, remove the percent sign and multiply by 0.01.



Note that 100% = 1.

100% = 100(0.01) = 1

Example 11

Write 130% as a fraction and as a decimal.

Solution:

$$130\% = 130\left(\frac{1}{100}\right)$$

$$= \frac{130}{100}$$

$$= 1\frac{3}{10}$$

To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

Example 11 – *Solution*

cont'd

$130\% = 130(0.01)$

$= 1.30$

To write a percent as a decimal, remove the percent sign and multiply by 0.01.

Example 12

Write $33\frac{1}{3}\%$ as a fraction.

Solution:

$$33\frac{1}{3}\% = 33\frac{1}{3}\left(\frac{1}{100}\right)$$

$$= \frac{100}{3}\left(\frac{1}{100}\right)$$

$$= \frac{1}{3}$$

Write the mixed number $33\frac{1}{3}$ as the improper fraction $\frac{100}{3}$.

Example 13

Write 0.25% as a decimal.

Solution:

$0.25\% = 0.25(0.01)$ Remove the percent sign and multiply by 0.01.

$= 0.0025$

Convert among percents, fractions, and decimals

A fraction or decimal can be written as a percent by multiplying by 100%.

Recall that 100% = 1, and multiplying a number by 1 does not change the value of the number.

To write $\frac{5}{8}$ as a percent, multiply by 100%.

$$\frac{5}{8} = \frac{5}{8}(100\%) = \frac{500}{8}\% = 62.5\% \text{ or } 62\frac{1}{2}\%$$

To write 0.82 as a percent, multiply by 100%.

$$0.82 = 0.82(100\%) = 82\%$$

Move the decimal point two places to the right. Then write the percent sign.

Example 14

Write as a percent. **A.** 0.027 **B.** 1.34

Solution:

A. 0.027 = 0.027(100%)

To write a decimal as a percent, multiply by 100%.

= 2.7%

B. 1.34 = 1.34(100%)

= 134%

Example 15

Write $\frac{5}{6}$ as a percent. Round to the nearest tenth of a percent.

Solution:

$$\frac{5}{6} = \frac{5}{6}(100\%)$$

To write a fraction as a percent, multiply by 100%.

$$= \frac{500}{6}\%$$

$$\approx 83.3\%$$

Example 16

Write $\frac{7}{16}$ as a percent. Write the remainder in fractional form.

Solution:

$$\frac{7}{16} = \frac{7}{16}(100\%)$$

Multiply the fraction by 100%.

$$= \frac{700}{16}\%$$

$$= 43\frac{3}{4}\%$$