Chapter 8

Locus and Concurrence

8.5 More Area Relationships in the Circle

More Area Relationships in the circle

Definition

A **sector** of a circle is a region bounded by two radii of the circle and an arc intercepted by those radii.
(See Figure 8.47.)

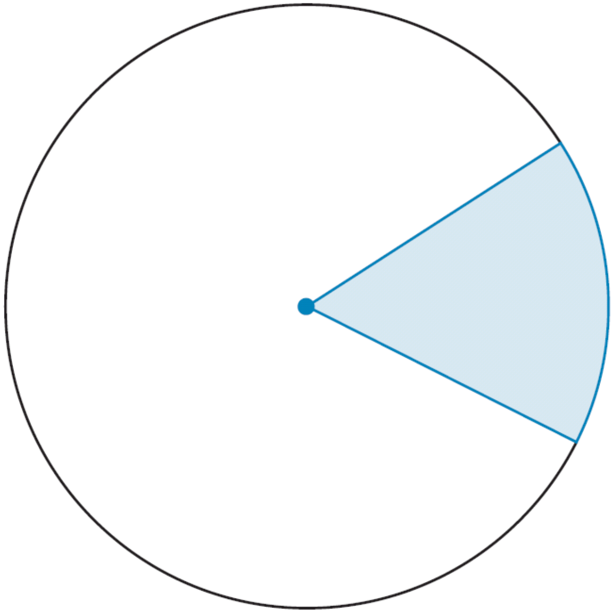


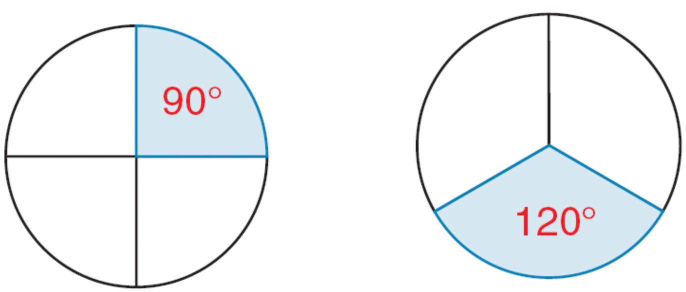
Figure 8.47

AREA OF A SECTOR

Area of a Sector

Just as the length of an arc is part of the circle's circumference, the area of a sector is part of the area of this circle.

When fractions are illustrated by using circles, $\frac{1}{4}$ is represented by shading a 90° sector, and $\frac{1}{3}$ is represented by shading a 120° sector (see Figure 8.48).



$\frac{1}{4} = \frac{90°}{360°}$ $\frac{1}{3} = \frac{120°}{360°}$

Figure 8.48

Area of a Sector

Postulate 23

The ratio of the degree measure *m* of the arc (or central angle) of a sector to 360° is the same as the ratio of the area of the sector to the area of the circle; that is,

$$\frac{\text{area of sector}}{\text{area or circle}} = \frac{m}{360}.$$

Theorem 8.5.1

In a circle of radius *r*, the area *A* of a sector whose arc has degree measure *m* is

$$A = \frac{m}{360}\pi r^2.$$

Example 1

If $m\angle O = 100°$, find the area of the 100° sector shown in Figure 8.49. Use your calculator and round the answer to the nearest hundredth of a square inch.

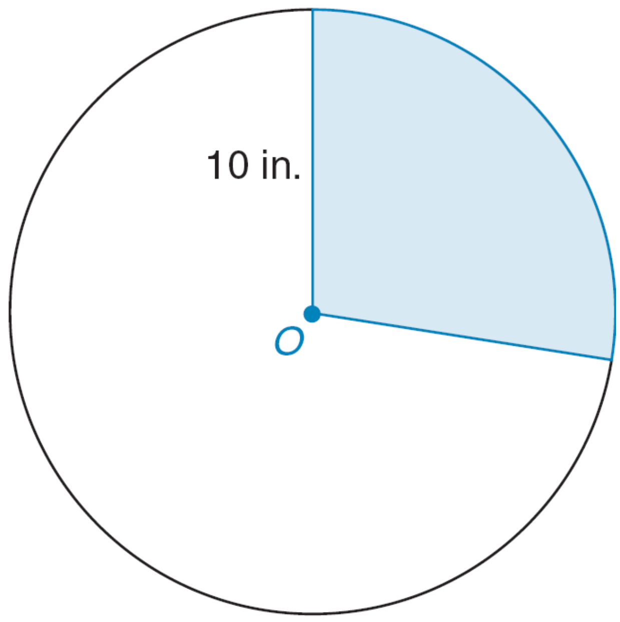


Figure 8.49

Example 1 – *Solution*

$$A = \frac{m}{360}\pi r^2$$

becomes

$$A = \frac{100}{360} \cdot \pi \cdot 10^2$$

$$\approx 87.27 \text{ in}^2$$

Area of a Sector

In applications with circles, we often seek exact answers for circumference and area; in such cases, we simply leave π in the result.

For instance, in a circle of radius length 5 in., the exact circumference is 10π in. and the exact area is expressed as 25π in^2.

Area of a Sector

Because a sector is bounded by two radii and an arc, the perimeter of a sector is the sum of the lengths of the two radii and the length of its arc.

Corollary 8.5.2

The area of a semicircular region of radius length r is

$A = \frac{1}{2}\pi r^2$.

AREA OF A SEGMENT

Area of a Segment

Definition

A **segment** of a circle is a region bounded by a chord and its minor (or major) arc.

In Figure 8.51, the segment is bounded by chord $\overline{AB}$ and its minor arc $\overset{\frown}{AB}$.

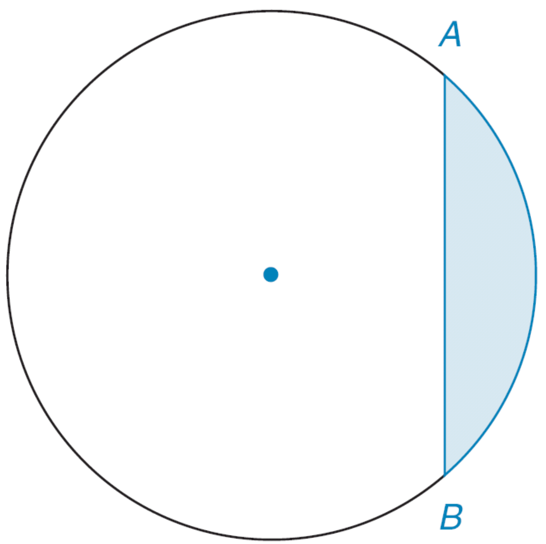


Figure 8.51

Example 4

Find the exact area of the segment bounded by the chord and the arc whose measure is 90°. The radius has length 12 in., as shown in Figure 8.52.

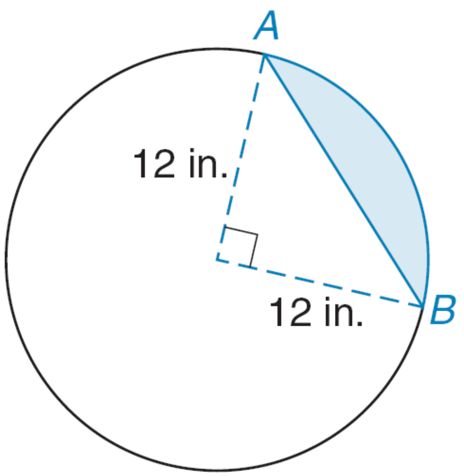


Figure 8.52

Example 4 – *Solution*

Let $A_{\triangle}$ represent the area of the triangle shown. Because

$$A_{\Delta} + A_{\text{segment}} = A_{\text{sector}}$$

we see that

$$A_{\text{segment}} = A_{\text{sector}} - A_{\Delta}$$

$$= \frac{90}{360} \cdot \pi \cdot 12^2 - \frac{1}{2} \cdot 12 \cdot 12$$

$$= \frac{1}{4} \cdot 144\pi - \frac{1}{2} \cdot 144$$

$$= (36\pi - 72) \text{ in}^2$$

Area of a Segment

In Example 4, the boundaries of the segment shown are chord $\overline{AB}$ and minor arc $\overset{\frown}{AB}$.

Therefore, the perimeter of the segment is given by

$$P_{\text{segment}} = AB + \ell\,\overset{\frown}{AB}$$

AREA OF A TRIANGLE WITH AN INSCRIBED CIRCLE

Area of a Triangle with an Inscribed Circle

Theorem 8.5.3

Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by

$$A = \frac{1}{2}rP$$

Example 6

Find the area of a triangle whose sides measure 5 cm, 12 cm, and 13 cm, if the radius of the inscribed circle is 2 cm. See Figure 8.55.

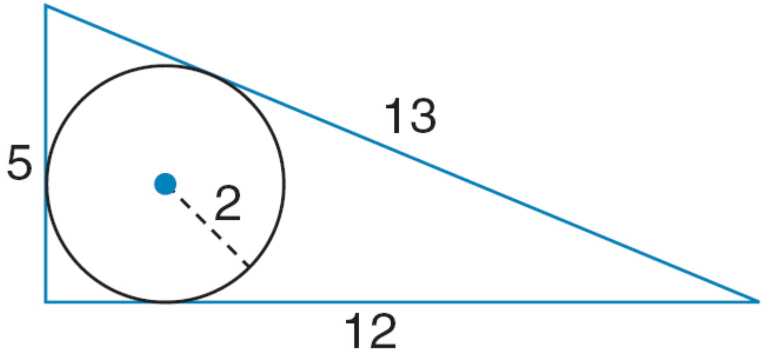


Figure 8.55

Example 6 – *Solution*

With the given lengths of sides, the perimeter of the triangle is

$$P = 5 + 12 + 13 = 30 \text{ cm.}$$

Using $A = \frac{1}{2}rP$,

we have $A = \frac{1}{2} \cdot 2 \cdot 30$ or

$$A = 30 \text{ cm}^2.$$

Area of a Triangle with an Inscribed Circle

Because the triangle shown in Example 6 is a right triangle ($5^2 + 12^2 = 13^2$), the area of the triangle could have been determined by using either

$$A = \frac{1}{2}ab \qquad \text{or} \qquad A = \sqrt{s(s-a)(s-b)(s-c)}.$$