Chapter 8

Areas of Polygons and Circles

8.3 Regular Polygons and Area

Regular Polygons and Area

Regular polygons are, of course, both equilateral and equiangular.

We can inscribe a circle within any regular polygon and we can circumscribe a circle about any regular polygon.

For regular hexagon *ABCDEF* shown in Figure 8.32, suppose that $\overline{QE}$ and $\overline{QD}$ bisect the interior angles of *ABCDEF* as shown.

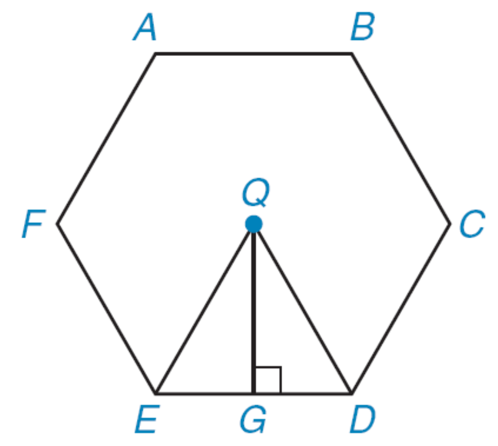

Figure 8.32

Regular Polygons and Area

1. Point *Q*, the *center* of regular hexagon *ABCDEF*, is the common center of both the inscribed and circumscribed circles for regular hexagon *ABCDEF*.

2. $\overline{QE}$ is a *radius* of regular hexagon *ABCDEF* because it joins the center of the regular polygon to a vertex.

3. $\overline{QG}$ is an *apothem* of regular hexagon *ABCDEF* because it is drawn from the center of the regular polygon perpendicular to a side of the polygon.

Regular Polygons and Area

4. $\angle EQD$ is a *central angle* of regular hexagon *ABCDEF* because center *Q* is the vertex of the central angle, while the sides are consecutive radii of the polygon. The measure of a central angle of a regular polygon of *n* sides is $c = \frac{360°}{n}$.

5. Any radius of a regular polygon bisects the interior angle to which it is drawn.

6. Any apothem of a regular polygon bisects the side to which it is drawn.

Regular Polygons and Area

Among regular polygons are the square and the equilateral triangle. The area of a square whose sides have length s is given by $A = s^2$.

Example 1

Find the area of the square whose apothem length is $a = 2$ in.

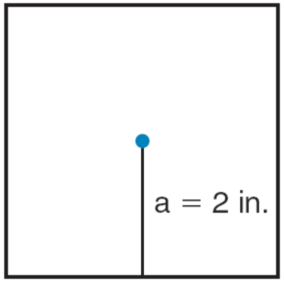

Figure 8.33

Solution:

The apothem is the perpendicular distance from the center to a side of length s.

For the square, $s = 2a$; that is, $s = 4$ in.

Then $A = s^2$ becomes $A = 4^2$ and $A = 16$ in^2.

AREA OF A REGULAR POLYGON

Area of a Regular Polygon

For a regular polygon of *n* sides, each side of length *s*, its perimeter is $P = n \cdot s$.

Theorem 8.3.1
The area *A* of a regular polygon whose apothem has length *a* and whose perimeter is *P* is given by

$$A = \frac{1}{2}aP$$

Example 4

Use $A = \frac{1}{2}aP$ to find the area of the square whose apothem length is a = 2 in.

Solution:

See figure 8.33. When the length of apothem of a square is a = 2, the length of side is s = 4.

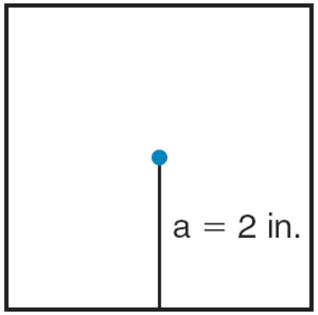

Figure 8.33

Example 4 – *Solution*

cont'd

In turn, the perimeter is $P = 16$ in.

Now $A = \frac{1}{2}aP$ becomes

$A = \frac{1}{2} \cdot 2 \cdot 16,$

so $A = 16 \text{ in}^2.$