Chapter 7

Locus and Concurrence

7.3 More About Regular Polygons

More about Regular Polygons

Several interesting properties of regular polygons are developed in this section.

For instance, every regular polygon has both an inscribed circle and a circumscribed circle; furthermore, these two circles are concentric.

In Example 1, we use bisectors of the angles of a square to locate the center of the inscribed circle.

The center, which is found by using the bisectors of any two *consecutive* angles, is equidistant from the sides of the square.

Example 1

Given square *ABCD* in Figure 7.26(a), construct inscribed $\odot O$.

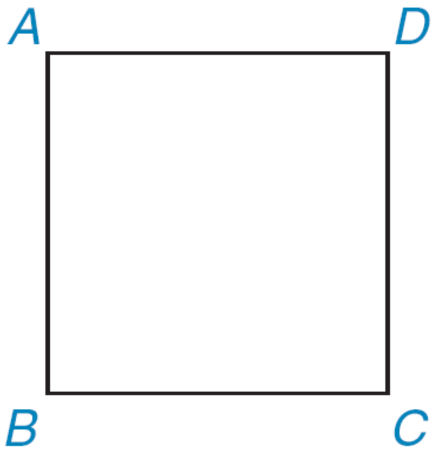

Figure 7.26(a)

Example 1 – *Solution*

Figure 7.26(b): The center of an inscribed circle must lie at the same distance from each side. Center *O* is the point of concurrency of the angle bisectors of the square.

Thus, we construct the angle bisectors of $\angle B$ and $\angle C$ to identify point *O*.

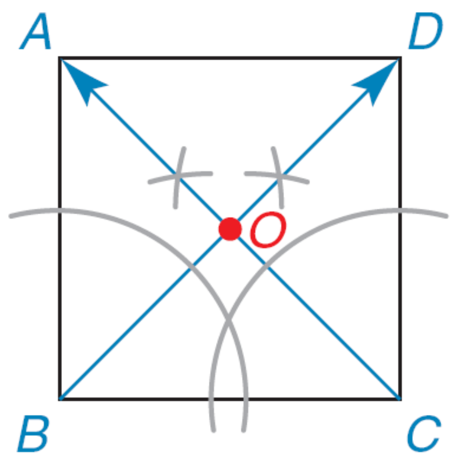

Figure 7.26(b)

Example 1 – *Solution*

cont'd

Figure 7.26(c): Constructing $\overline{OM} \perp \overline{AB}$, *OM* is the distance from *O* to $\overline{AB}$ and the length of the radius of the inscribed circle.

Finally we construct inscribed $\odot O$ with radius $\overline{OM}$ as shown.

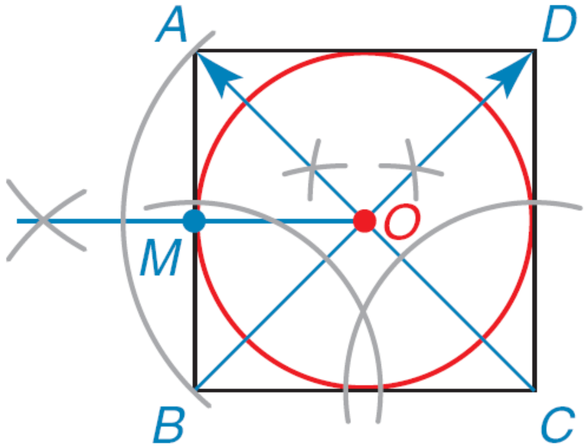

Figure 7.26(c)

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For a rectangle, which is not a regular polygon, we can only circumscribe a circle (see Figure 7.28).

For a rhombus (also not a regular polygon), we can only inscribe a circle (see Figure 7.29).

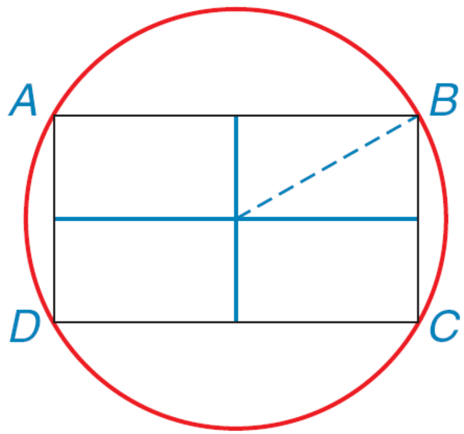

Figure 7.28

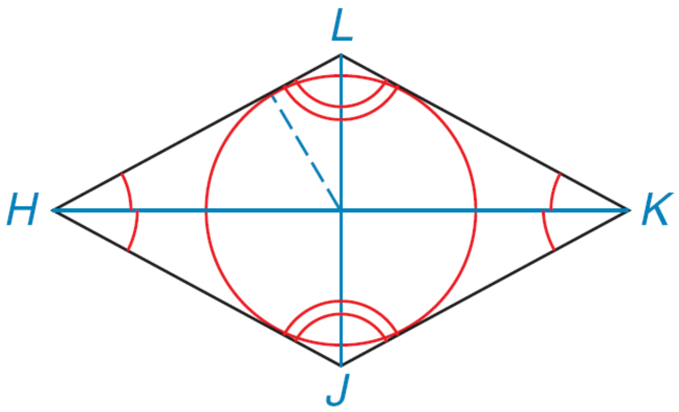

Figure 7.29

More about Regular Polygons

As we shall see, we can construct both inscribed and circumscribed circles for regular polygons because they are both equilateral and equiangular.

A few of the regular polygons are shown in Figure 7.30.

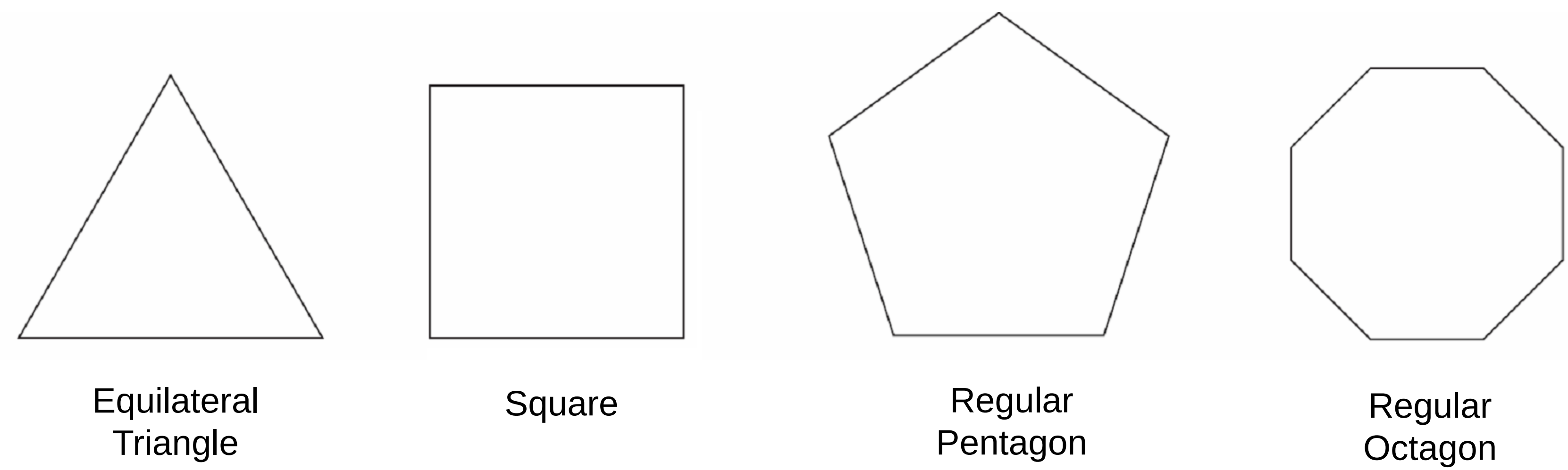

Figure 7.30

More about Regular Polygons

In the following table, we recall these facts.

TABLE 7.2

Regular Polygons (n sides)

	Interior Angles	**Exterior Angles**
Sum	$(n-2)\cdot 180°$	$360°$
Each Angle	$\dfrac{(n-2)\cdot 180°}{n}$	$\dfrac{360°}{n}$

The number of diagonals is $D = \dfrac{n(n-3)}{2}$.

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Regular polygons allow us to inscribe and to circumscribe a circle. The proof of the following theorem will establish the following relationships:

1. The centers of the inscribed and circumscribed circles of a regular polygon are the same.

2. The angle bisectors of two consecutive angles or the perpendicular bisectors of two consecutive sides can be used to locate the common center of the inscribed circle and the circumscribed circle.

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3. The inscribed circle’s radius is any line segment from the center drawn perpendicular to a side of the regular polygon; also, the radius of the circumscribed circle joins the center to any vertex of the regular polygon.

Theorem 7.3.1

A circle can be circumscribed about (or inscribed in) any regular polygon.

More about Regular Polygons

Definition

The **center of a regular polygon** is the common center for the inscribed and circumscribed circles of the polygon.

In Figure 7.32, point *O* is the center of the regular pentagon *RSTVW*. In this figure, $\overline{OR}$ is called a "radius" of the regular pentagon because it is the radius of the circumscribed circle.

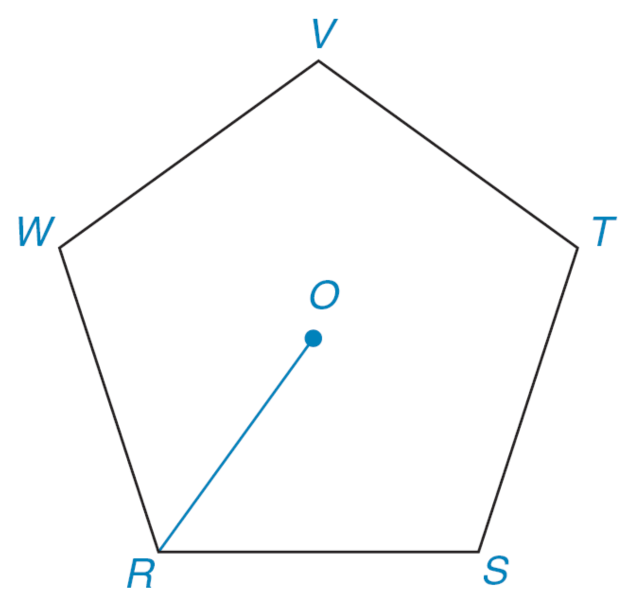

Figure 7.32

More about Regular Polygons

Definition

A **radius of a regular polygon** is any line segment that joins the center of the regular polygon to one of its vertices.

More about Regular Polygons

Definition

An **apothem** of a regular polygon is any line segment drawn from the center of that polygon perpendicular to one of the sides.

In regular octagon *RSTUVWXY* with center *P* in Figure 7.33, the segment $\overline{PQ}$ is an apothem. An apothem of a regular polygon is a radius of the inscribed circle. Any regular polygon of *n* sides has *n* apothems and *n* radii.

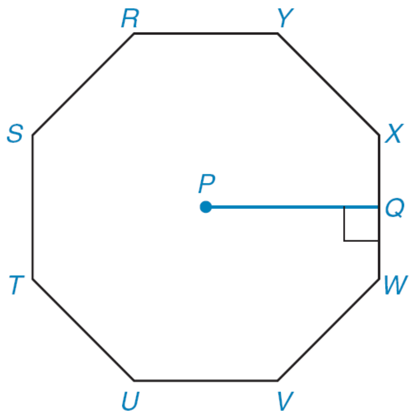

Figure 7.33

More about Regular Polygons

Definition

A **central angle of a regular polygon** is an angle formed by two consecutive radii of the regular polygon.

In regular hexagon *ABCDEF* with center *Q* (see Figure 7.34), angle *EQD* is a central angle.

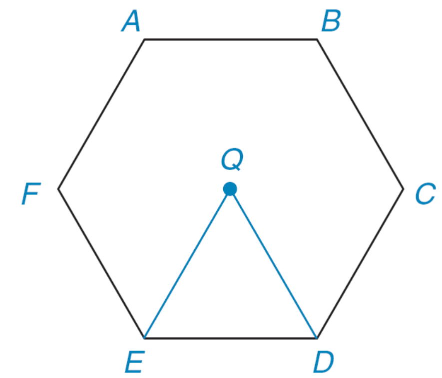

Figure 7.34

Theorem 7.3.2

The measure of any central angle of a regular polygon of *n* sides is given by $c = \frac{360}{n}$.

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Theorem 7.3.3

Any radius of a regular polygon bisects the angle at the vertex to which it is drawn.

Theorem 7.3.4

Any apothem of a regular polygon bisects the side of the polygon to which it is drawn.