Chapter 7 Locus and Concurrence

7.1 Locus of Points

Locus of Points

In some instances, we describe the set of points whose locations satisfy a given condition or set of conditions.

Definition

A **locus** is the set of all points and only those points that satisfy a given condition (or set of conditions).

Locus of Points

In this definition, the phrase “all points and only those points” has a dual meaning:

1. All points of the locus satisfy the given condition.

2. All points satisfying the given condition are included in the locus.

The set of points satisfying a given locus can be a well-known geometric figure such as a line or a circle. In Example 1, several points are located in a plane and then connected in order to form the locus.

Example 1

Describe the locus of points in a plane that are at a fixed distance (r) from a given point (P).

Solution:

See Figure 7.1. Each point shown in red is the same distance r from point P.

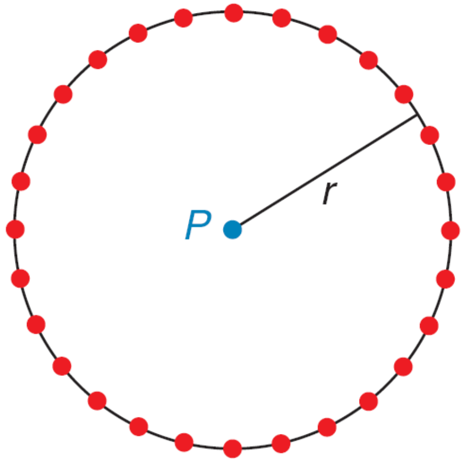

Figure 7.1

Example 1 – *Solution*

cont'd

Thus, the locus of points at fixed distance *r* from point *P* is the circle with center *P* and radius length *r*.

Locus of Points

Some definitions are given in a locus format; for example, the following is an alternative definition of the term **circle**.

Definition

A **circle** is the locus of points in a plane that are at a fixed distance from a given point.

The preceding example includes the phrase “in a plane.” If that phrase is omitted, the locus is found “in space.”

Locus of Points

For instance, the locus of points that are at a fixed distance from a given point is actually a *sphere* (the three-dimensional object in Figure 7.4); the sphere has the fixed point as center, and the fixed distance determines the length of the radius.

Unless otherwise stated, we will consider the locus to be restricted to a plane.

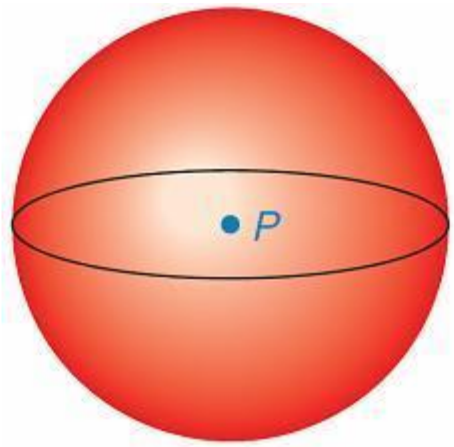

Figure 7.4

Locus of Points

Following are two very important theorems involving the locus concept. When we verify the locus theorems, we *must* establish two results:

1. If a point is in the locus, then it satisfies the condition.

2. If a point satisfies the condition, then it is a point of the locus.

Locus of Points

Theorem 7.1.1

The locus of points in a plane and equidistant from the sides of an angle is the angle bisector.

In locus problems, we must remember to demonstrate two relationships in order to validate results.

Locus of Points

A second important theorem regarding a locus of points follows.

Theorem 7.1.2

The locus of points in a plane that are equidistant from the endpoints of a line segment is the perpendicular bisector of that line segment.

Locus of Points

We now return to further considerations of a locus in a plane. Suppose that a given line segment is to be used as the hypotenuse of a right triangle.

How might one locate possible positions for the vertex of the right angle? One method might be to draw 30° and 60° angles at the endpoints so that the remaining angle formed must measure 90° [see Figure 7.9(a)].

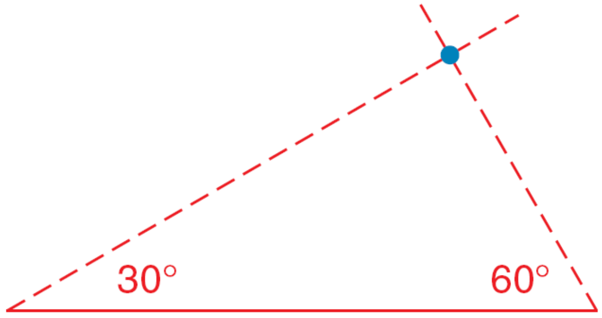

Figure 7.9(a)

Locus of Points

This is only one possibility, but because of symmetry, it actually provides four permissible points, which are indicated in Figure 7.9(b).

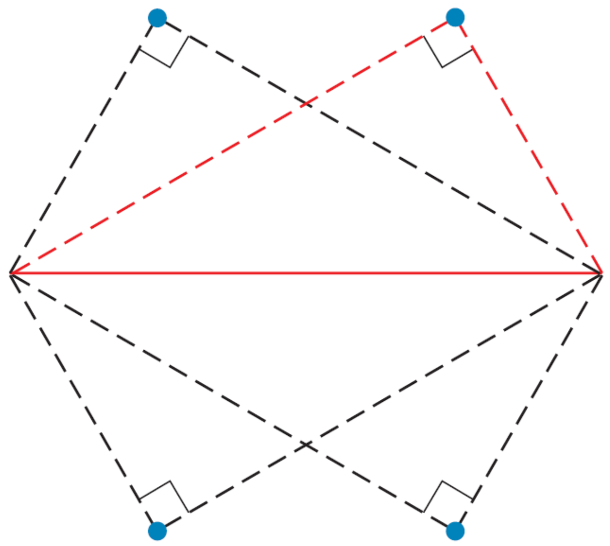

Figure 7.9(b)