Chapter 6

Circles

6.4 Some Constructions and Inequalities for the Circle

Some Constructions and Inequalities for the Circle

Recall that there is only one line perpendicular to a given line at a point on that line.

Theorem 6.4.1

The line that is perpendicular to the radius of a circle at its endpoint on the circle is a tangent to the circle.

CONSTRUCTIONS OF TANGENTS TO CIRCLES

Constructions of Tangents to Circles

Construction 8

To construct a tangent to a circle at a point on the circle.

Given: $\odot P$ with point X on the circle [See Figure 6.55(a).]

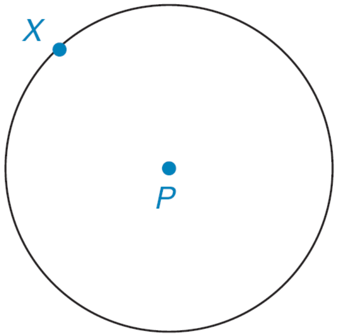

Figure 6.55(a)

Construct: A tangent $\overleftrightarrow{XW}$ to $\odot$ P at point X

Constructions of Tangents to Circles

Plan: The strategy used in Construction 8 is based on Theorem 6.4.1.

Construction: Figure 6.55(a): Consider $\odot P$ and point X on $\odot P$.

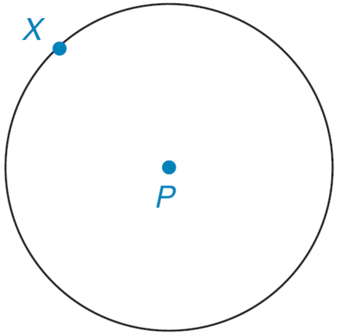

Figure 6.55(a)

Constructions of Tangents to Circles

Figure 6.55(b): Draw radius $\overline{PX}$ and extend it to form $\overrightarrow{PX}$. Using *X* as the center and any radius length less than *XP*, draw two arcs to intersect $\overrightarrow{PX}$ at points *Y* and *Z*.

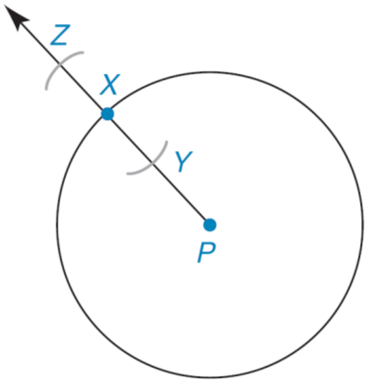

Figure 6.55(b)

Constructions of Tangents to Circles

Figure 6.55(c): Complete the construction of the line perpendicular to $\overrightarrow{PX}$ at point *X*. From *Y* and *Z*, mark arcs with equal radii have a length greater than *XY*.

Calling the point of intersection *W*, draw $\overleftrightarrow{XW}$, the desired tangent to $\odot P$ at point *X*.

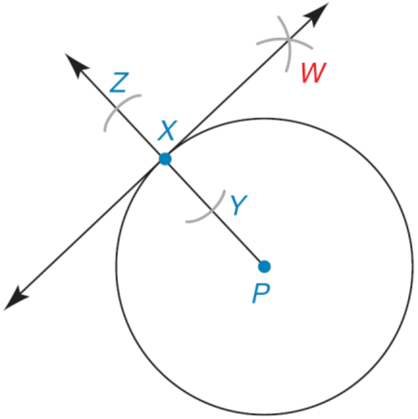

Figure 6.55 (c)

Example 1

Make a drawing so that points *A, B, C,* and *D* are on $\odot$ *O* in that order. If tangents are constructed at points *A, B, C,* and *D*, what type of quadrilateral will be formed by the tangent segments if

a) $m\overset{\frown}{AB} = m\overset{\frown}{CD}$ and $m\overset{\frown}{BC} = m\overset{\frown}{AD}$?

b) all arcs $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, and $\overset{\frown}{DA}$ are congruent?

Solution:

a) A rhombus (all sides are congruent)

b) A square (all four $\angle$s are right $\angle$s; all sides $\cong$)

Constructions of Tangents to Circles

Construction 9

To construct a tangent to a circle from an external point.

Given: $\odot Q$ and external point *E* [See Figure 6.56(a).]

Figure 6.56(a)

Construct: A tangent $\overline{ET}$ to $\odot Q$, with *T* as the point of tangency

Constructions of Tangents to Circles

Construction: Figure 6.56(a): Consider $\odot Q$ and external point E.

Figure 6.56(b): Draw $\overline{EQ}$. Construct the perpendicular bisector of $\overline{EQ}$, to intersect $\overline{EQ}$ at its midpoint M.

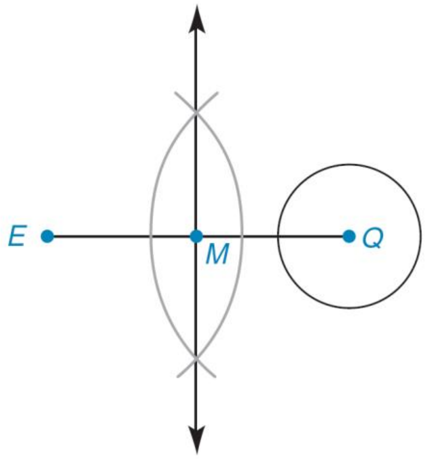

Figure 6.56(b)

Constructions of Tangents to Circles

Figure 6.56(c): With *M* as center and *MQ* (or *ME*) as the length of radius, construct a circle. The points of intersection of circle *M* with circle *Q* are designated by *T* and *V*.

Now draw $\overline{ET}$, the desired tangent.

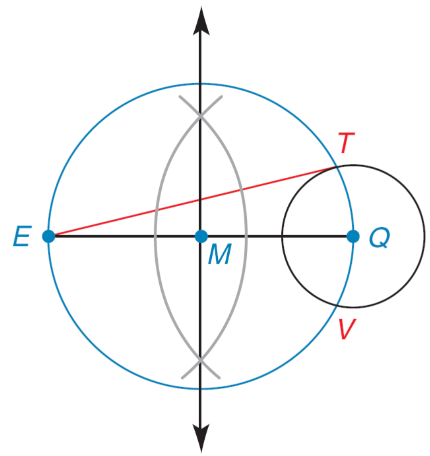

Figure 6.56(c)

Constructions of Tangents to Circles

In the preceding construction, $\overline{QT}$ (not shown) is a radius of the smaller circle Q.

In the larger circle M, $\angle ETQ$ is an inscribed angle that intercepts a semicircle.

Thus, $\angle ETQ$ is a right angle and $\overline{ET} \perp \overline{TQ}$.

Because the line ($\overline{ET}$) drawn perpendicular to the radius ($\overline{TQ}$) of a circle at its endpoint on the circle is a tangent to the circle, $\overline{ET}$ is a tangent to circle Q.

INEQUALITIES IN THE CIRCLE

Inequalities in the Circle

The remaining theorems in this section involve inequalities in the circle.

Theorem 6.4.2

In a circle (or in congruent circles) containing two unequal central angles, the larger angle corresponds to the larger intercepted arc.

Inequalities in the Circle

The converse of Theorem 6.4.2 follows, and it is also easily proved.

Theorem 6.4.3

In a circle (or in congruent circles) containing two unequal arcs, the larger arc corresponds to the larger central angle.

Example 2

Given: In Figure 6.58, $\odot Q$ with $m\overset{\frown}{RS} > m\overset{\frown}{TV}$.

a) Using Theorem 6.4.3, what conclusion can you draw regarding the measures of $\angle RQS$ and $\angle TQV$?

b) What does intuition suggest regarding RS and TV?

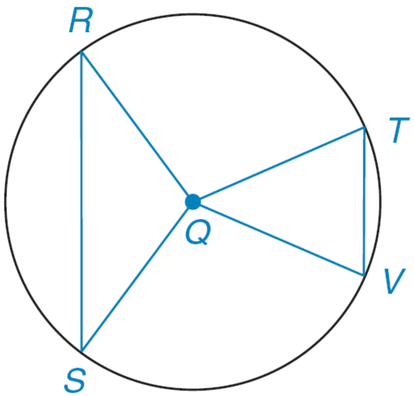

Figure 6.58

Example 2 – *Solution*

a) $m\angle RQS > m\angle TQV$

b) $RS > TV$

Inequalities in the Circle

Theorem 6.4.4

In a circle (or in congruent circles) containing two unequal chords, the shorter chord is at the greater distance from the center of the circle.

Theorem 6.4.5

In a circle (or in congruent circles) containing two unequal chords, the chord nearer the center of the circle has the greater length.

Inequalities in the Circle

Theorem 6.4.6

In a circle (or in congruent circles) containing two unequal chords, the longer chord corresponds to the greater minor arc.

If $AB > CD$ in Figure 6.61, then $m\overset{\frown}{AB} > m\overset{\frown}{CD}$.

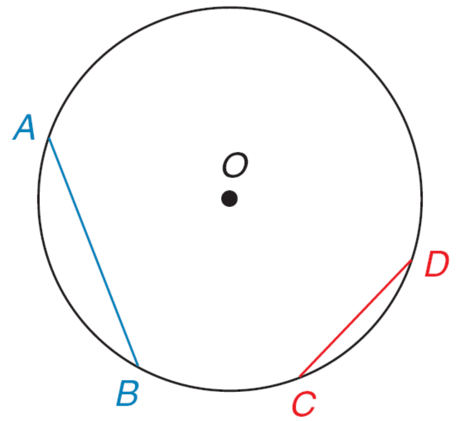

Figure 6.61

Inequalities in the Circle

Theorem 6.4.7

In a circle (or in congruent circles) containing two unequal minor arcs, the greater minor arc corresponds to the longer of the chords related to these arcs.