Chapter 6 Circles

6.3 Line and Segment Relationships in the Circle

Line and Segment Relationships in the Circle

Theorem 6.3.1

If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc.

Theorem 6.3.2

If a line through the center of a circle bisects a chord other than a diameter, then it is perpendicular to the chord.

Line and Segment Relationships in the Circle

Figure 6.39 (a) illustrates the following theorem.

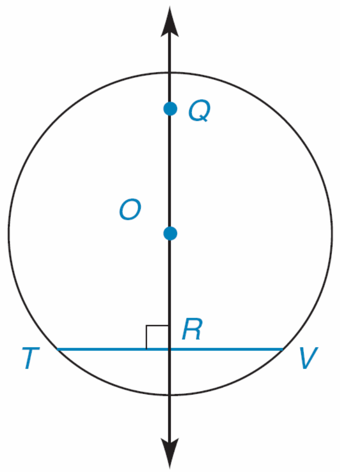

Figure 6.39 (a)

Theorem 6.3.3

The perpendicular bisector of a chord contains the center of the circle.

Example 1

Given: In Figure 6.40, $\odot$ *O* has a radius of length 5

$\overline{OE} \perp \overline{CD}$ at *B* and *OB* = 3

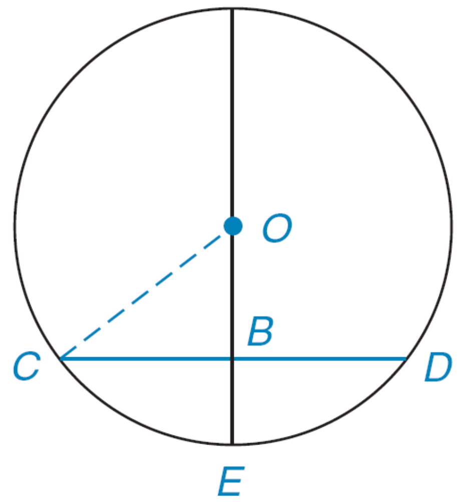

Figure 6.40

Find: *CD*

Example 1 – *Solution*

Draw radius $\overline{OC}$. By the Pythagorean Theorem,

$$(OC)^2 = (OB)^2 + (BC)^2$$

$$5^2 = 3^2 + (BC)^2$$

$$25 = 9 + (BC)^2$$

$$(BC)^2 = 16$$

$$BC = 4$$

We know that $CD = 2 \cdot BC$; then it follows that $CD = 2 \cdot 4 = 8$.

CIRCLES THAT ARE TANGENT

Circles that are Tangent

In this section, we assume that two circles are coplanar.

Although concentric circles do not intersect, they do share a common center.

For the concentric circles shown in Figure 6.41, the tangent of the smaller circle is a chord of the larger circle.

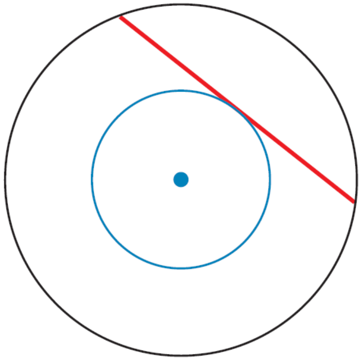

Figure 6.41

Circles that are Tangent

If two circles touch at one point, they are **tangent circles.**

In Figure 6.42(a), circles *P* and *Q* are **internally tangent**; in Figure 6.42(b), circles *O* and *R* are **externally tangent.**

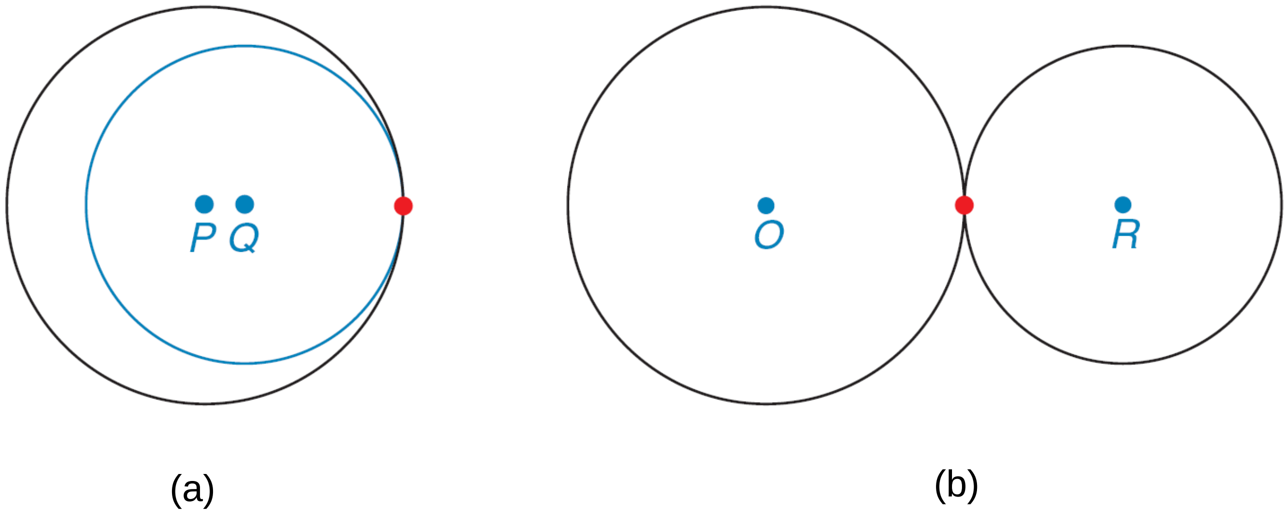

Figure 6.42

Circles that are Tangent

Definition

For two circles with different centers, the **line of centers** is the line (or line segment) containing the centers of both circles.

As the definition suggests, the line segment joining the centers of two circles is also commonly called the line of centers of the two circles.

In Figure 6.43, $\overleftrightarrow{AB}$ or $\overline{AB}$ is the line of centers for circles *A* and *B*.

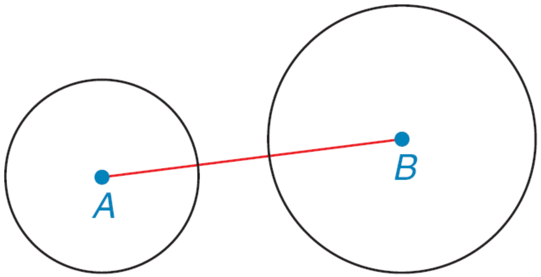

Figure 6.43

COMMON TANGENT LINES TO CIRCLES

Common Tangent Lines to Circles

A line, line segment, or ray that is tangent to each of two circles is a **common tangent** for these circles.

If the common tangent *does not* intersect the line segment joining the centers, it is a **common external tangent.**

Common Tangent Lines to Circles

In Figure 6.44(a), circles *P* and *Q* have one common external tangent, $\overleftrightarrow{ST}$; in Figure 6.44(b), circles *A* and *B* have two common external tangents, $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$.

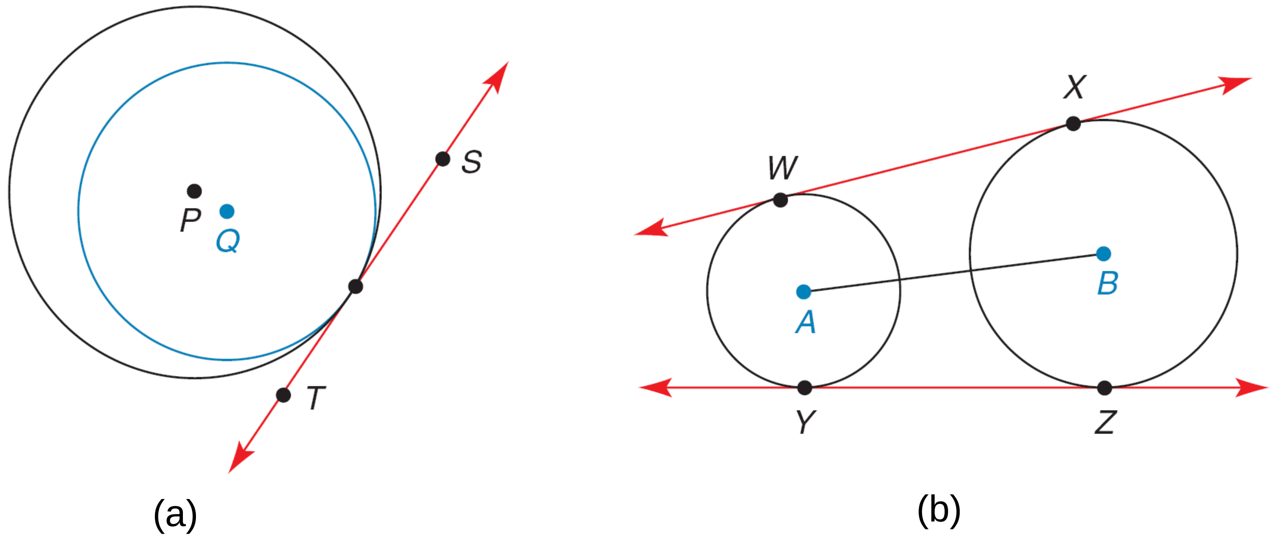

Figure 6.44

Common Tangent Lines to Circles

If the common tangent for two circles *does* intersect the line of centers for these circles, it is a **common internal tangent** for the two circles.

In Figure 6.45 (a), $\overleftrightarrow{DE}$ is a common internal tangent for externally tangent circles *O* and *R*; in Figure 6.45(b), $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are common internal tangents for $\odot M$ and $\odot N$.

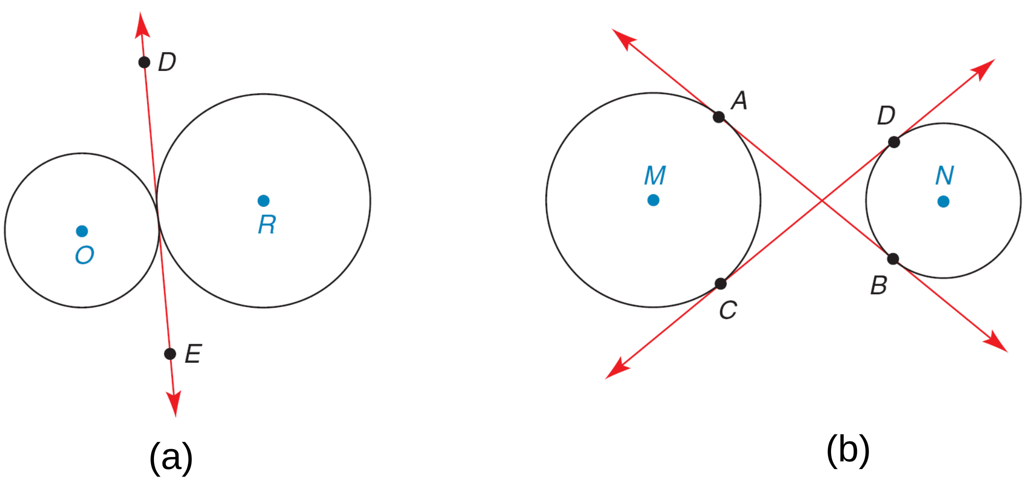

Figure 6.45

Common Tangent Lines to Circles

Theorem 6.3.4

The tangent segments to a circle from an external point are congruent.

Example 2

A belt used in an automobile engine wraps around two pulleys with different lengths of radii. Explain why the straight pieces named $\overline{AB}$ and $\overline{CD}$ have the same length.

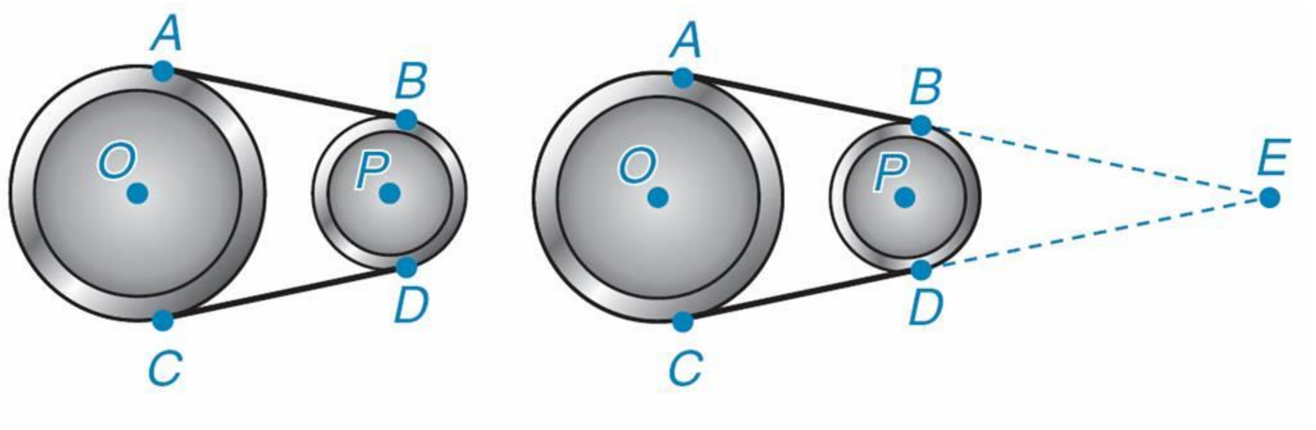

Figure 6.47

Example 2 – *Solution*

Pulley centered at O has the larger radius length, so we extend $\overline{AB}$ and $\overline{CD}$ to meet at point E.

Because E is an external point to both $\odot O$ and $\odot P$, we know that $EB = ED$ and $EA = EC$ by Theorem 6.3.4.

By subtracting equals from equals, $EA - EB = EC - ED$.

Because $EA - EB = AB$ and $EC - ED = CD$, it follows that $AB = CD$.

LENGTHS OF LINE SEGMENTS IN A CIRCLE

Lengths of Line Segments in a Circle

To complete this section, we consider three relationships involving the lengths of chords, secants, or tangents.

Theorem 6.3.5

If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.

Example 4

In Figure 6.50, $HP = 4$, $PJ = 5$, and $LP = 8$. Find PM.

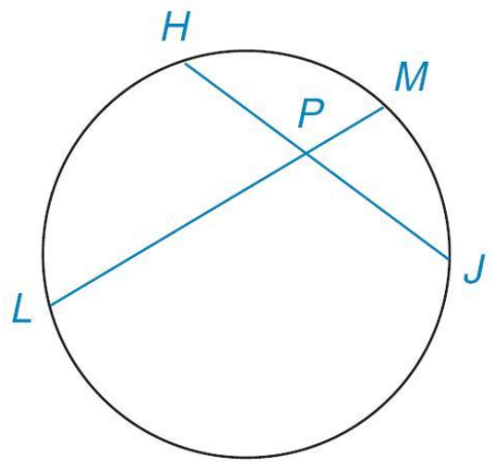

Figure 6.50

Solution:

Applying Theorem 6.3.5, we have

$$HP \cdot PJ = LP \cdot PM.$$

Then

$$4 \cdot 5 = 8 \cdot PM$$

$$8 \cdot PM = 20$$

$$PM = 2.5$$

Lengths of Line Segments in a Circle

Theorem 6.3.6

If two secant segments are drawn to a circle from an external point, then the products of the length of each secant with the length of its external segment are equal.

Theorem 6.3.7

If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.