Chapter 5

Similar Triangles

5.5 Special Right Triangles

THE 45°-45°-90° RIGHT TRIANGLE

The 45°-45°-90° Right Triangle

In the 45**°**-45**°**-90**°** triangle, the legs lie opposite the congruent angles and are also congruent.

Rather than using *a* and *b* to represent the lengths of the legs, we use *a* for both lengths; see Figure 5.31.

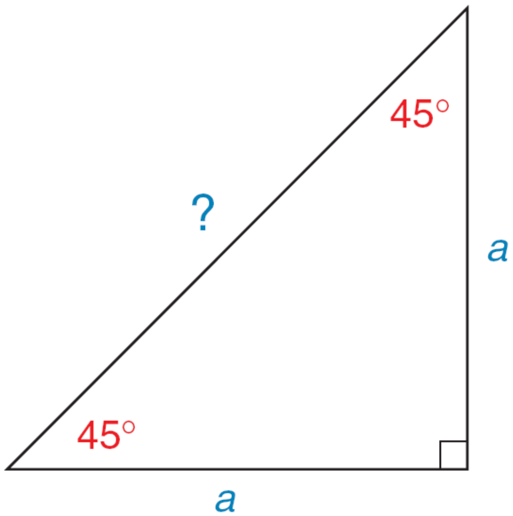


Figure 5.31

The 45°-45°-90° Right Triangle

By the Pythagorean Theorem, it follows that

$$c^2 = a^2 + a^2$$

$$c^2 = 2a^2$$

$$c = \sqrt{2a^2}$$

$$c = \sqrt{2} \cdot \sqrt{a^2}$$

$$c = a\sqrt{2}$$

The 45°-45°-90° Right Triangle

Theorem 5.5.1 (45-45-90 Theorem)

In a right triangle whose angles measure 45°, 45°, and 90°, the legs are congruent and the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

Example 1

Find the lengths of the missing sides in each triangle in Figure 5.33.

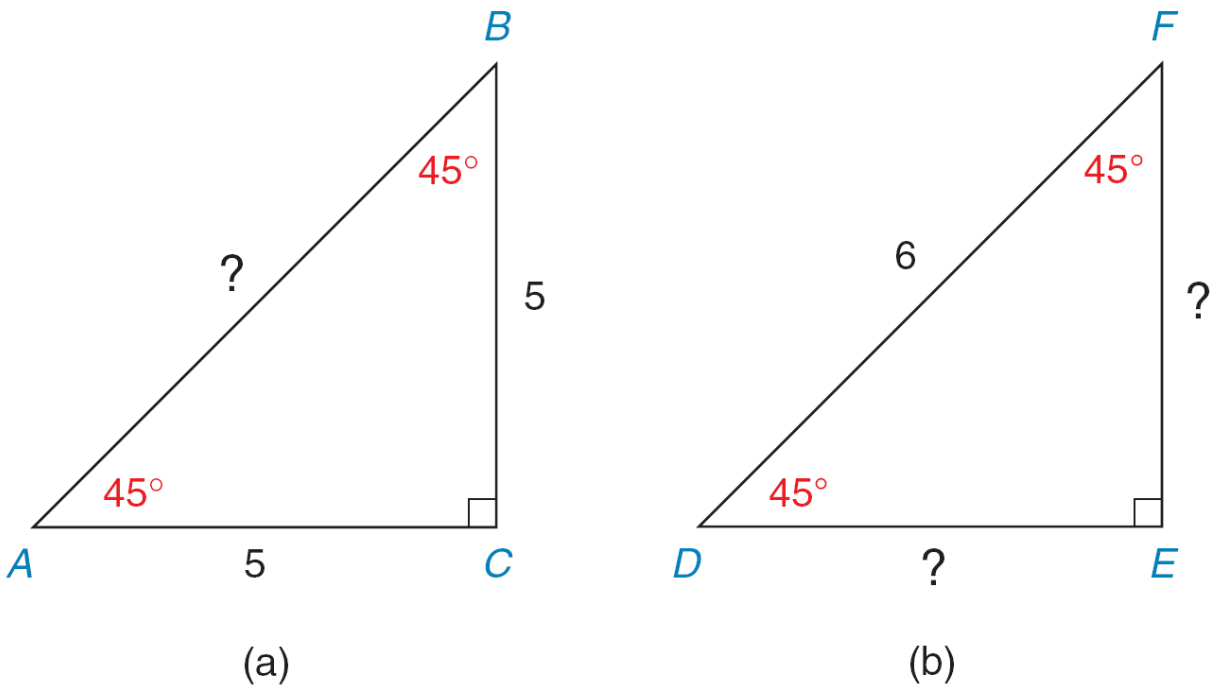


Figure 5.33

Example 1 – *Solution*

a) The length of hypotenuse $\overline{AB}$ is $5\sqrt{2}$, the product of $\sqrt{2}$ and the length of either of the equal legs.

b) Let *a* denote the length of $\overline{DE}$ and of $\overline{EF}$. The length of hypotenuse $\overline{DF}$ is $a\sqrt{2}$.

Then $a\sqrt{2}$ = 6, so *a* = $\frac{6}{\sqrt{2}}$.

Simplifying yields $$a = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

Example 1 – *Solution*

cont'd

$$= \frac{6\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$

Therefore, *DE* = *EF* = $3\sqrt{2} \approx 4.24$.

THE 30°-60°-90° RIGHT TRIANGLE

The 30°-60°-90° Right Triangle

The second special triangle is the 30°-60°-90° triangle.

Theorem 5.5.2 (30-60-90 Theorem)

In a right triangle whose angles measure 30°, 60°, and 90°, the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

Example 3

Study the picture proof of Theorem 5.5.2.
See Figure 5.35(a).

Picture Proof Of Theorem 5.5.2

Given: $\triangle ABC$ with m$\angle A = 30°$**,**

m$\angle B = 60°$**,** m$\angle C = 90°$**,** and $BC = a$

Prove: $AB = 2a$ and $AC = a\sqrt{3}$

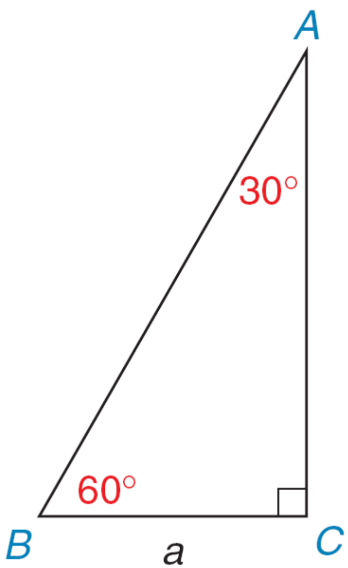


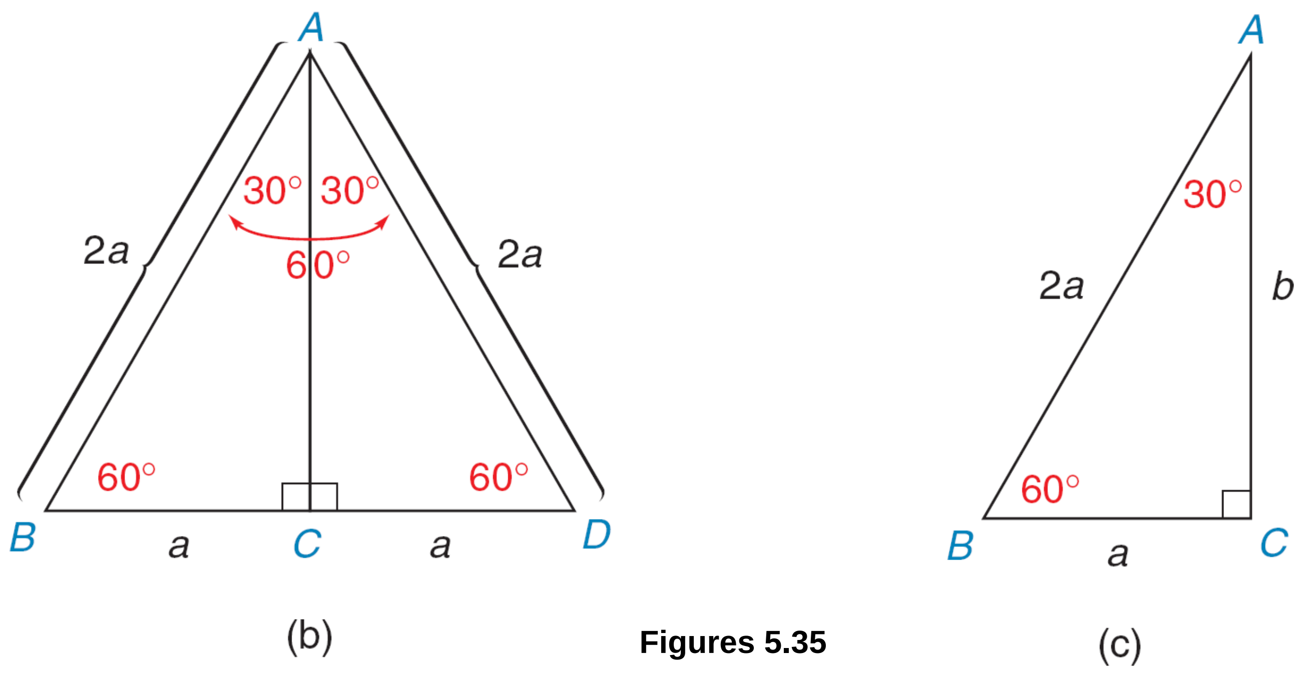
Figure 5.35(a)

Example 3

Proof:

We reflect $\triangle ABC$ across $\overline{AC}$ to form an equiangular and therefore equilateral $\triangle ABD$.

As shown in Figures 5.35(b) and 5.35(c), we have $AB = 2a$.



Figures 5.35

Example 3

cont'd

To find b in Figure 5.35(c), we apply the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

$$(2a)^2 = a^2 + b^2$$

$$4a^2 = a^2 + b^2$$

$$3a^2 = b^2$$

Example 3

cont'd

So

$$b^2 = 3a^2$$

$$b = \sqrt{3a^2}$$

$$b = \sqrt{3} \cdot \sqrt{a^2}$$

$$b = a\sqrt{3}$$

That is, $AC = a\sqrt{3}$

The 30°-60°-90° Right Triangle

It would be best to memorize the sketch in Figure 5.36.

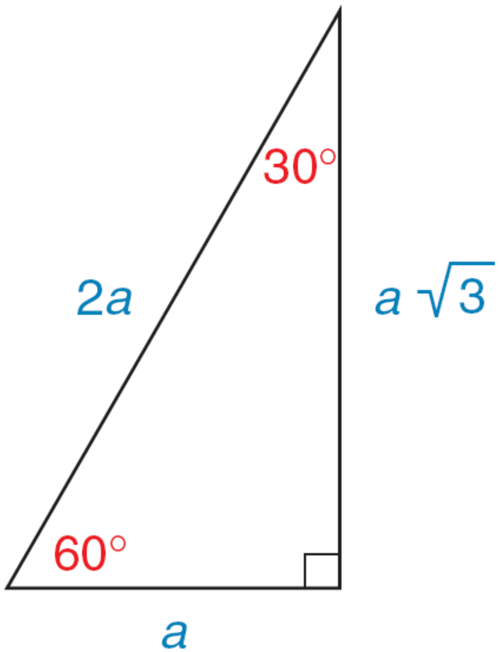


Figure 5.36

So that you will more easily recall which expression is used for each side, remember that the lengths of the sides follow the same order as the angles opposite them.

The 30°-60°-90° Right Triangle

Thus,

THE 30°-60°-90° TRIANGLE		
Angle Measure		**Length of Opposite Side**
30°	$\longleftrightarrow$	a
60°	$\longleftrightarrow$	$a\sqrt{3}$
90°	$\longleftrightarrow$	$2a$

The 30°-60°-90° Right Triangle

Theorem 5.5.3

If the length of the hypotenuse of a right triangle equals the product of $\sqrt{2}$ and the length of either congruent leg, then the angles of the triangle measure 45°, 45°, and 90°.

The 30°-60°-90° Right Triangle

Theorem 5.5.4

If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30°.

An equivalent form of this theorem is stated as follows:

> If one leg of a right triangle has a length equal to one-half the length of the hypotenuse, then the angle of the triangle opposite that leg measures 30° (see Figure 5.42).

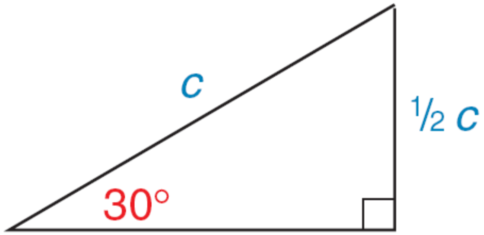


Figure 5.42