Chapter 5

Similar Triangles

5.3 Proving Triangles Similar

Proving Triangles Similar

It is quite difficult to establish a proportionality between the lengths of the corresponding sides of polygons. For this reason, our definition of similar polygons (and therefore of similar triangles) is almost impossible to use as a method of proof.

Fortunately, some easier methods are available for proving triangles similar. If two triangles are carefully sketched or constructed so that their angles are congruent, the triangles will have the same shape as shown in Figure 5.11.

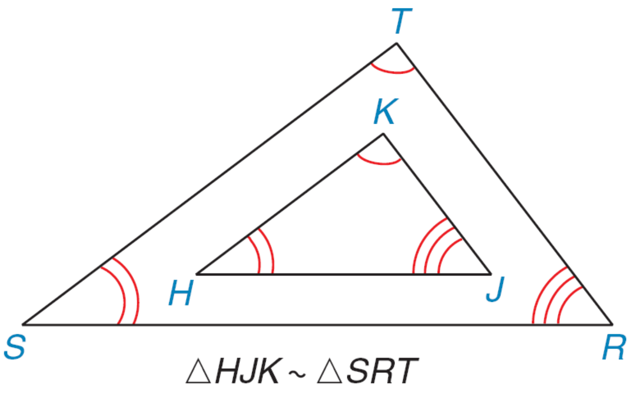

Figure 5.11

Proving Triangles Similar

Postulate 15

If the three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar (AAA).

Corollary 5.3.1 of Postulate 15 follows from knowing that if two angles of one triangle are congruent to two angles of another triangle, then the third angles *must* also be congruent.

Proving Triangles Similar

Corollary 5.3.1

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar (AA).

Rather than use AAA to prove triangles similar, we will use AA instead because it requires fewer steps.

Example 1

Provide a two-column proof of the following problem.

Given: $\overline{AB} \parallel \overline{DE}$ in Figure 5.12

Prove: $\triangle ABC \sim \triangle EDC$

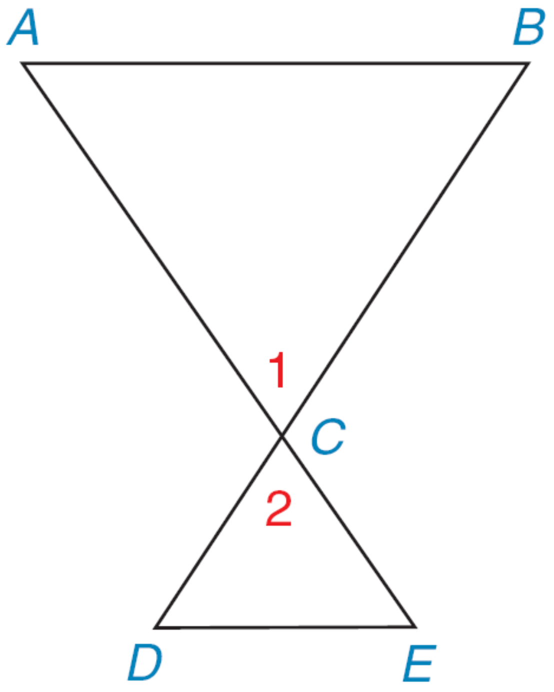

Figure 5.12

Example 1 cont'd

Proof:

Statements	Reasons
1. $\overline{AB} \parallel \overline{DE}$	1. Given
2. $\angle A \cong \angle E$	2. If two || lines are cut by a transversal, the alternate interior angles are ≅
3. $\angle 1 \cong \angle 2$	3. Vertical angles are ≅
4. $\triangle ABC \sim \triangle EDC$	4. AA

Proving Triangles Similar

In some instances, we wish to prove a relationship that takes us beyond the similarity of triangles.

The following consequences of the definition of similarity are often cited as reasons in a proof.

CSSTP

Corresponding sides of similar triangles are proportional.

CASTC

Corresponding angles of similar triangles are congruent.

Proving Triangles Similar

Although the CSSTP statement involves triangles, the corresponding sides of *any* two similar polygons are proportional.

That is, the ratio of the lengths of any pair of corresponding sides of one polygon equals the ratio of the lengths of another pair of corresponding sides of the second polygon.

Proving Triangles Similar

Theorem 5.3.2

The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

Proving Triangles Similar

In addition to AA, there are other methods that can be used to establish similar triangles.

To distinguish the following techniques for showing triangles similar from methods for proving that triangles are congruent, we use SAS~ and SSS~ to identify the similarity theorems.

Theorem 5.3.3 (SAS~)

If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including the angles are proportional, then the triangles are similar.

Proving Triangles Similar

Theorem 5.3.4 (SSS~)

If the three sides of one triangle are proportional to the three corresponding sides of a second triangle, then the triangles are similar.

Along with AA and SAS~, Theorem 5.3.4 (SSS~) provides the third (and final) method of establishing that triangles are similar.

Proving Triangles Similar

Lemma 5.3.5

If a line segment divides two sides of a triangle proportionally, then this line segment is parallel to the third side of the triangle.