Chapter 5

Similar Triangles

5.2 Similar Polygons

Similar Polygons

When two geometric figures have exactly the same shape, they are **similar;** the symbol for “is similar to” is **~**.

When two figures have the same shape (~) and all corresponding parts have equal (=) measures, the two figures are **congruent** (≅).

Note that the symbol for congruence combines the symbols for similarity and equality; that is, congruent polygons always have the same shape and the measures of corresponding parts are equal.

Two congruent polygons are also similar polygons.

Similar Polygons

Two-dimensional figures such as $\triangle ABC$ and $\triangle DEF$ in Figure 5.3 can be similar, it is also possible for three-dimensional figures to be similar.

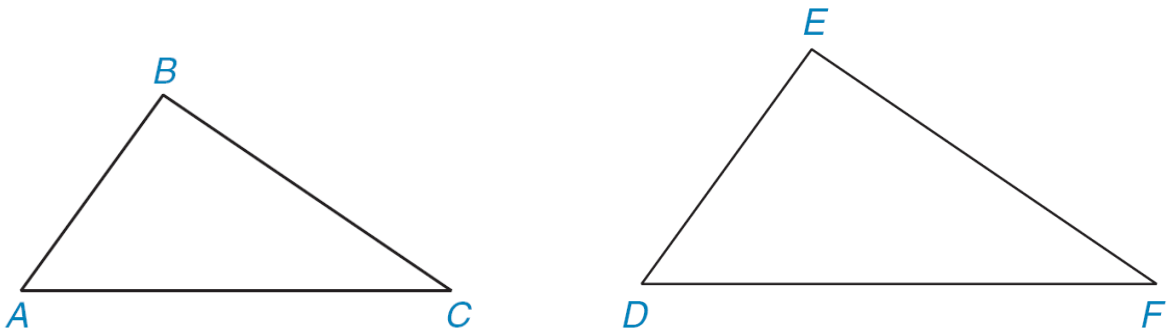
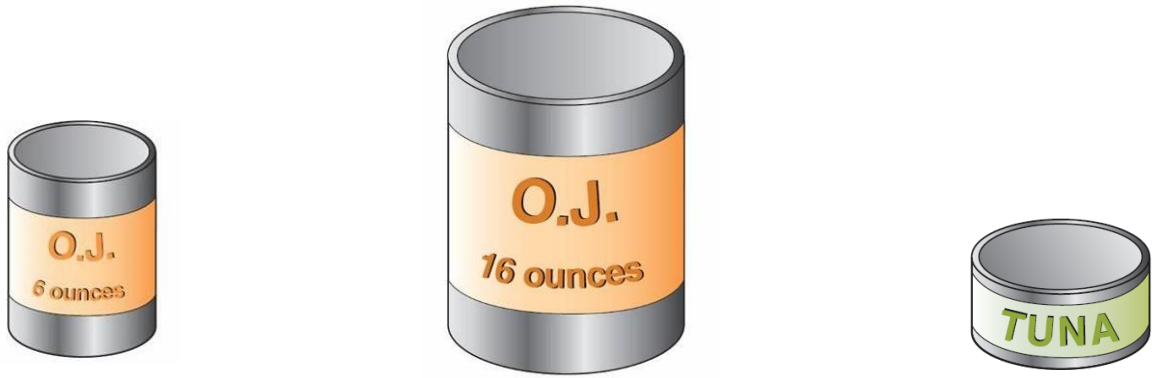


Figure 5.3

Similar Polygons

Similar orange juice containers are shown in Figures 5.4 (a) and 5.4 (b). Informally, two figures are "similar" if one is an enlargement of the other.

Thus a tuna fish can and an orange juice can are *not* similar, even if both are right-circular cylinders [see Figures 5.4(b) and 5.4(c)].



(a) (b) (c)

Figure 5.4

Similar Polygons

Our discussion of similarity will generally be limited to plane figures.

For two polygons to be similar, it is necessary that each angle of one polygon be congruent to the corresponding angle of the other.

However, the congruence of angles is not sufficient to establish the similarity of polygons.

The vertices of the congruent angles are **corresponding vertices** of the similar polygons.

Similar Polygons

If $\angle A$ in one polygon is congruent to $\angle H$ in the second polygon, then vertex *A* corresponds to vertex *H,* and this is symbolized $A \leftrightarrow H$; we can indicate that $\angle A$ corresponds to $\angle H$ by writing $\angle A \leftrightarrow \angle H$.

A pair of angles like $\angle A$ and $\angle H$ are **corresponding angles,** and the sides determined by consecutive and corresponding vertices are **corresponding sides** of the similar polygons.

For instance, if $A \leftrightarrow H$ and $B \leftrightarrow J$, then $\overline{AB}$ corresponds to $\overline{HJ}$.

Example 1

Given similar quadrilaterals *ABCD* and *HJKL* with congruent angles as indicated in Figure 5.5, name the vertices, angles, and sides that correspond to each other.

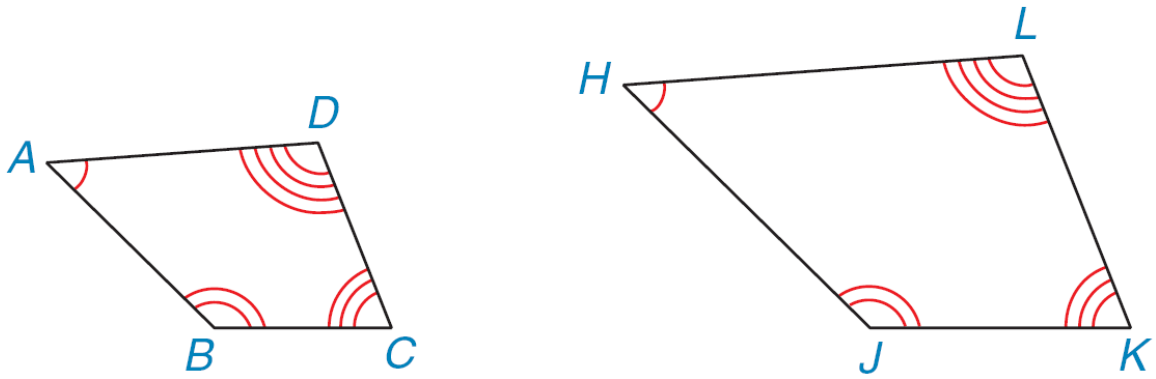


Figure 5.5

Example 1 – *Solution*

Because $\angle A \cong \angle H$, it follows that

$A \leftrightarrow H$ and $\angle A \leftrightarrow \angle H$

Similarly,

$B \leftrightarrow J$ and $\angle B \leftrightarrow \angle J$

$C \leftrightarrow K$ and $\angle C \leftrightarrow \angle K$

D $\leftrightarrow$ L and $\angle D \leftrightarrow \angle L$

Example 1 – *Solution*

cont'd

Associating pairs of consecutive and corresponding vertices of similar polygons, we determine the endpoints of the corresponding sides.

$$\overline{AB} \leftrightarrow \overline{HJ}, \quad \overline{BC} \leftrightarrow \overline{JK}, \quad \overline{CD} \leftrightarrow \overline{KL}, \quad \text{and} \quad \overline{AD} \leftrightarrow \overline{HL}$$

Similar Polygons

With an understanding of the terms corresponding angles and corresponding sides, we can define similar polygons.

Definition

Two polygons are **similar** if and only if two conditions are satisfied:

1. All pairs of corresponding angles are congruent.

2. All pairs of corresponding sides are proportional.

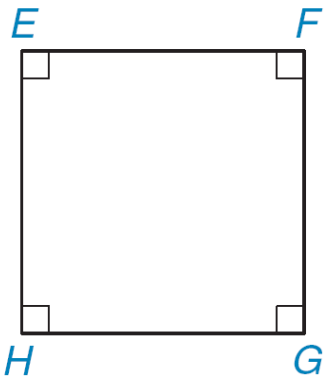
Similar Polygons

The second condition for similarity requires that the following extended proportion exists for the sides of the similar quadrilaterals of Example 1.

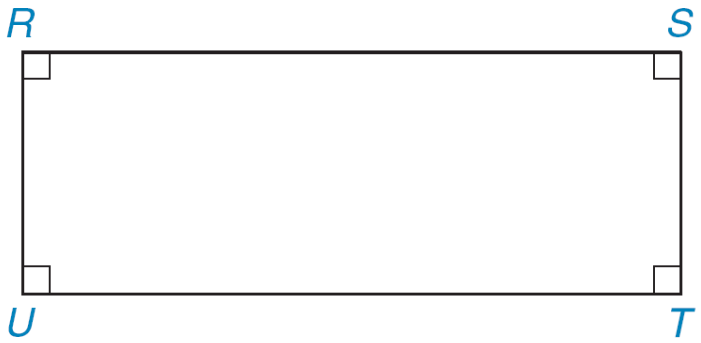
$$\frac{AB}{HJ} = \frac{BC}{JK} = \frac{CD}{KL} = \frac{AD}{HL}$$

Similar Polygons

Note that *both* conditions 1 and 2 for similarity are necessary! Although condition 1 is satisfied for square *EFGH* and rectangle *RSTU* [see Figures 5.6(a) and (b)], the figures are not similar. That is, one is not an enlargement of the other because the extended proportion comparing the lengths of corresponding sides is not true.



(a)

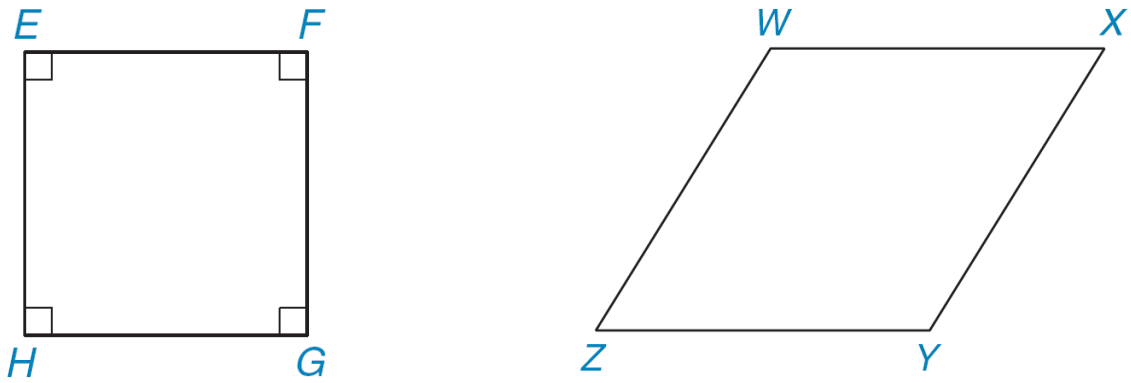


(b)

Figure 5.6

Similar Polygons

On the other hand, condition 2 is satisfied for square *EFGH* and rhombus *WXYZ* [see Figures 5.6(a) and 5.6(c)], but the figures are not similar because the pairs of corresponding angles are not congruent.



(a) (c)

Figure 5.6