Chapter 4

Quadrilaterals

4.4 The Trapezoid

The Trapezoid

Definition

A **trapezoid** is a quadrilateral with exactly two parallel sides.

Consider Figure 4.32. If $\overline{HL} \parallel \overline{JK}$, then *HJKL* is a trapezoid.

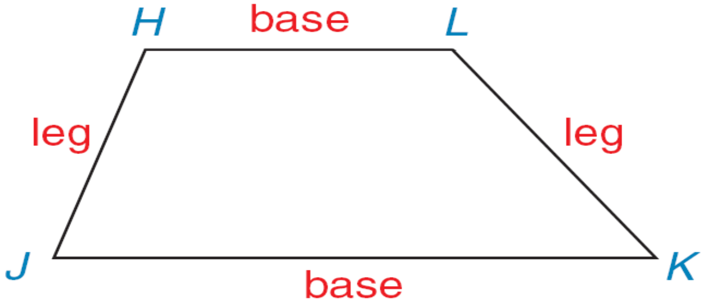

Figure 4.32

The Trapezoid

The parallel sides $\overline{HL}$ and $\overline{JK}$ of trapezoid *HJKL* are its **bases,** and the nonparallel sides $\overline{HJ}$ and $\overline{LK}$ are its **legs.**

Because $\angle J$ and $\angle K$ both have $\overline{JK}$ for a side, they are a pair of **base angles** of the trapezoid; $\angle H$ and $\angle L$ are also a pair of base angles because $\overline{HL}$ is a base.

When the midpoints of the two legs of a trapezoid are joined, the resulting line segment is known as the **median** of the trapezoid.

The Trapezoid

Given that *M* and *N* are the midpoints of the legs $\overline{HJ}$ and $\overline{LK}$ in trapezoid *HJKL*, $\overline{MN}$ is the median of the trapezoid. [See Figure 4.33(a)].

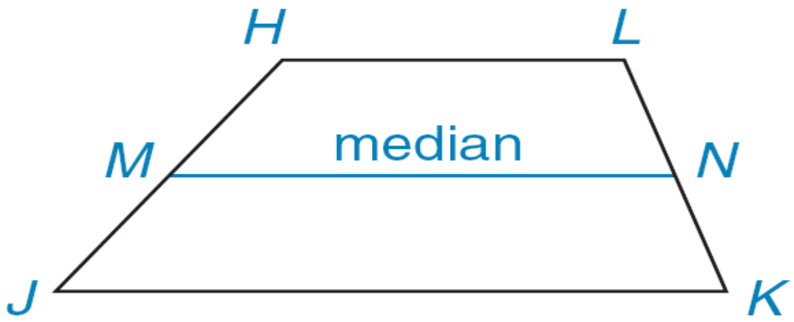

Figure 4.33(a)

The Trapezoid

If the two legs of a trapezoid are congruent, the trapezoid is known as an **isosceles trapezoid.**

In Figure 4.33(b), *RSTV* is an **isosceles trapezoid** because $\overline{RS} \parallel \overline{VT}$ and $\overline{RV} \cong \overline{ST}$.

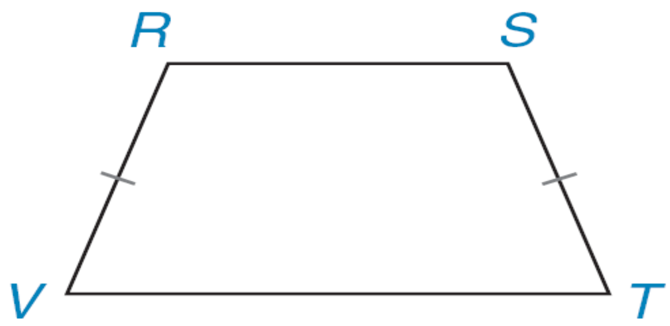

Figure 4.33(b)

The Trapezoid

Every trapezoid contains two pairs of consecutive interior angles that are supplementary.

Each of these pairs of angles is formed when parallel lines are cut by a transversal.

In Figure 4.33(c), angles *H* and *J* are supplementary, as are angles *L* and *K*.

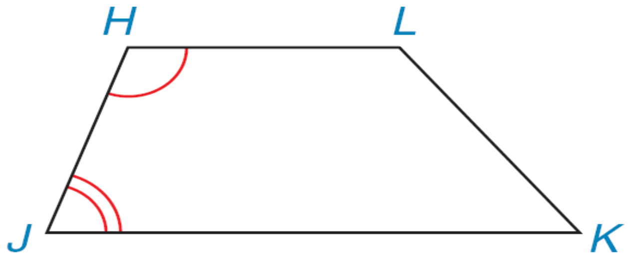

Figure 4.33(c)

The Trapezoid

Given that $\overline{WX} \parallel \overline{ZY}$ and that $\angle Z$ is a right angle in Figure 4.33(d), trapezoid *WXYZ* is a **right trapezoid**. Based upon the "Reminder," we conclude that $\angle W$ is a right angle as well.

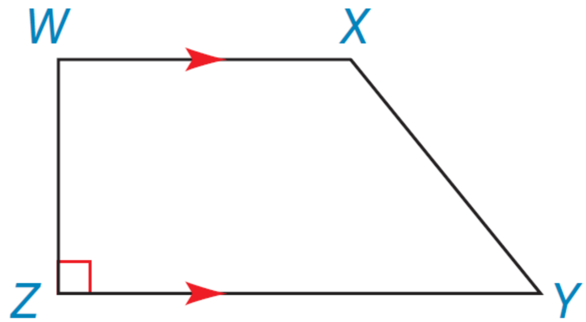

Figure 4.33(d)

Example 1

In Figure 4.32, suppose that m$\angle H$ = 107° and m$\angle K$ = 58°.

Find m$\angle J$ and m$\angle L$.

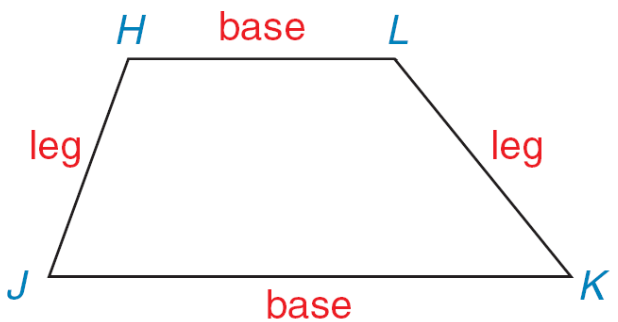

Figure 4.32

Example 1 – *Solution*

Because $\overline{HL} \parallel \overline{JK}$, $\angle$s *H* and *J* are supplementary angles, as are $\angle$s *L* and *K*.

Then m$\angle H$ + m $\angle J$ = 180 and m$\angle L$ + m$\angle K$ = 180.

Substitution leads to 107 + m$\angle J$ = 180 and m$\angle L$ + 58 = 180, so m$\angle J$ = 73° and m$\angle L$ = 122°.

The Trapezoid

Definition

An **altitude** of a trapezoid is a line segment from one base of the trapezoid perpendicular to the opposite base (or to an extension of that base).

In Figure 4.34, $\overline{HX}$, $\overline{LY}$, $\overline{JP}$, and $\overline{KQ}$ are altitudes of trapezoid *HJKL*.

The length of any altitude of *HJKL* is called the *height* of the trapezoid.

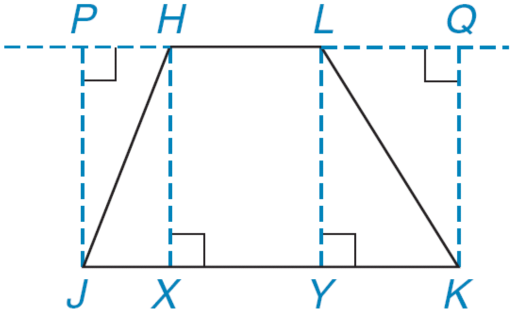

Figure 4.34

The Trapezoid

Theorem 4.4.1

The base angles of an isosceles trapezoid are congruent.

The following statement is a corollary of Theorem 4.4.1.

Corollary 4.4.2

The diagonals of an isosceles trapezoid are congruent.

The Trapezoid

Theorem 4.4.3

The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.

Theorem 4.4.4

The median of a trapezoid is parallel to each base.

The Trapezoid

Theorems 4.4.5 and 4.4.6 enable us to show that a trapezoid with certain characteristics is an isosceles trapezoid. We state these theorems as follows:

Theorem 4.4.5

If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

The Trapezoid

Theorem 4.4.6

If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

Theorem 4.4.7

If three (or more) parallel lines intercept congruent line segments on one transversal, then they intercept congruent line segments on any transversal.