Chapter 4 Quadrilaterals

4.3 The Rectangle, Square, and Rhombus

THE RECTANGLE

The Rectangle

In this section, we investigate special parallelograms. The first of these is the rectangle (symbol ▭ and abbreviated "rect."), which is defined as follows:

Definition

A **rectangle** is a parallelogram that has a right angle. (See Figure 4.20.)

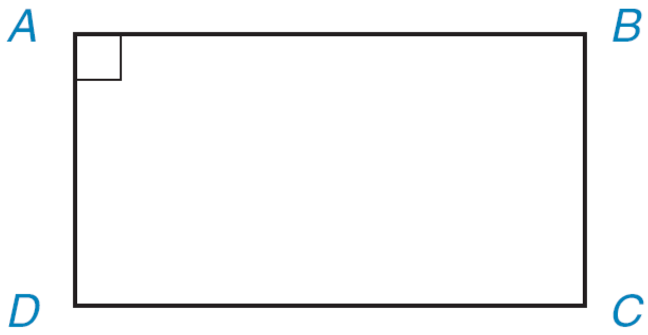


Figure 4.20

The Rectangle

Any reader who is familiar with the rectangle may be confused by the fact that the preceding definition calls for only one right angle.

Because a rectangle is a parallelogram by definition, the fact that a rectangle has four right angles is easily proved by applying Corollaries 4.1.3 and 4.1.5.

Corollary 4.1.3

The opposite sides of a parallelogram are congruent.

Corollary 4.1.5

Two consecutive angles of a parallelogram are supplementary.

The Rectangle

Corollary 4.3.1

All angles of a rectangle are right angles.

The following theorem is true for rectangles, but not for parallelograms in general.

Corollary 4.3.2

The diagonals of a rectangle are congruent.

Example 1

Complete a proof of Theorem 4.3.2. Use Figure 4.21.

Given: ▭*MNPQ* with diagonals $\overline{MP}$ and $\overline{NQ}$

Prove: $\overline{MP} \cong \overline{NQ}$

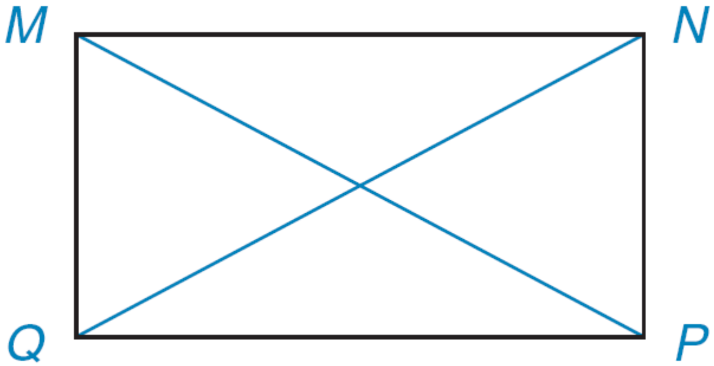


Figure 4.21

Example 1

cont'd

Proof:

Statements	Reasons
1. ▭ *MNPQ* with diagonals $\overline{MP}$ and $\overline{NQ}$	1. Given
2. *MNPQ* is a ▱	2. By definition, a rectangle is a ▱ with a right angle
3. $\overline{MN} \cong \overline{QP}$	3. Opposite sides of a ▱ are $\cong$
4. $\overline{MQ} \cong \overline{MQ}$	4. Identity

Example 1

cont'd

Statements	Reasons
5. $\angle NMQ$ and $\angle PQM$ are right $\angle$s	5. By Corollary 4.3.1, the four $\angle$s of a rectangle are right $\angle$s
6. $\angle NMQ \cong \angle PQM$	6. All right $\angle$s are $\cong$
7. $\triangle NMQ \cong \triangle PQM$	7. SAS
8. $\overline{MP} \cong \overline{NQ}$	8. CPCTC

THE SQUARE

The Square

All rectangles are parallelograms; some parallelograms are rectangles; and some rectangles are *squares*.

Definition

A **square** is a rectangle that has two congruent adjacent sides. (See Figure 4.22.)

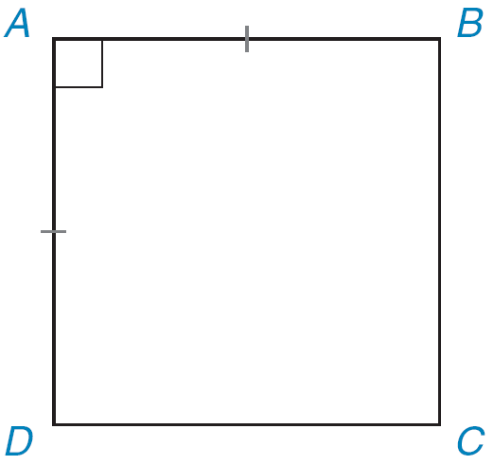


Figure 4.22

The Square

Corollary 4.3.3

All sides of a square are congruent.

Because a square is a type of rectangle, it has four right angles and its diagonals are congruent. Because a square is also a parallelogram, its opposite sides are parallel. For any square, we can show that the diagonals are perpendicular.

We measure area in “square units.”

THE RHOMBUS

The Rhombus

The next type of quadrilateral we consider is the rhombus. The plural of the word *rhombus* is *rhombi* (pronounced rhŏm-bī).

Definition

A **rhombus** is a parallelogram with two congruent adjacent sides.

In Figure 4.23, the adjacent sides $\overline{AB}$ and $\overline{AD}$ of rhombus *ABCD* are marked congruent.

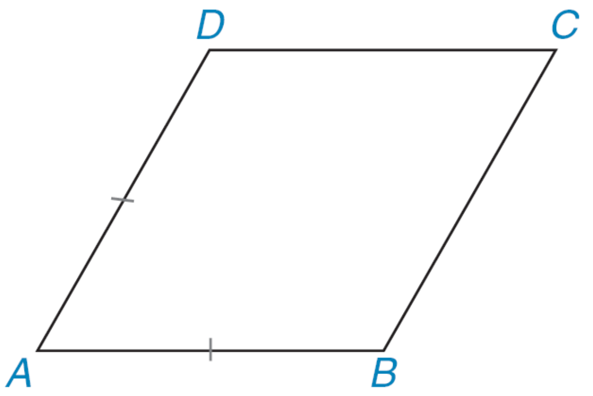


Figure 4.23

The Rhombus

Because a rhombus is a type of parallelogram, it is also necessary that $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$.
Thus, we have Corollary 4.3.4.

Corollary 4.3.4

All sides of a rhombus are congruent.

We will use Corollary 4.3.4 in the proof of the following theorem.

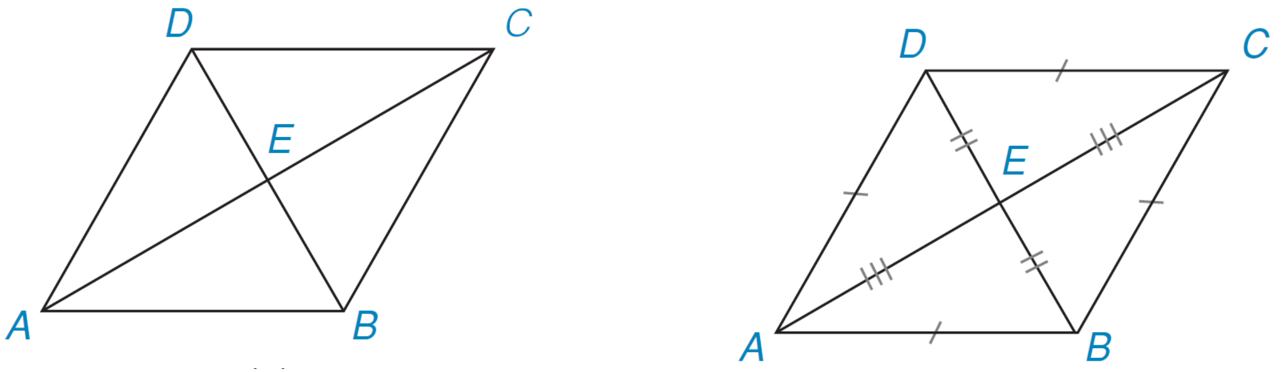
Theorem 4.3.5

The diagonals of a rhombus are perpendicular.

Example 2

Study the picture proof of Theorem 4.3.5. In the proof, pairs of triangles are congruent by the reason SSS.

Picture Proof of Theorem 4.3.5



(a) (b)

Figure 4.24

Example 2

cont'd

Given: Rhombus *ABCD*, with diagonals $\overline{AC}$ and $\overline{DB}$ [See Figure 4.24(a)].

Prove: $\overline{AC} \perp \overline{DB}$

Proof:

Rhombus *ABCD* has congruent sides as indicated. The diagonals of rhombus *ABCD* (a type of parallelogram) bisect each other.

By SSS, the four small triangles are congruent; thus, each angle at vertex *E* is a right angle. Then $\overline{AC} \perp \overline{DB}$.

The Rhombus

An alternative definition of *square* is “A square is a rhombus whose adjacent sides form a right angle.” Therefore, a further property of a square is that its diagonals are perpendicular.

Because the square and the rhombus are both types of parallelograms, we have the following consequence.

Corollary 4.3.6

The diagonals of a rhombus (or square) are perpendicular bisectors of each other.

THE PYTHAGOREAN THEOREM

The Pythagorean Theorem

The Pythagorean Theorem, which deals with right triangles, is also useful in applications involving quadrilaterals that have right angles. In antiquity, the theorem claimed that "the square upon the hypotenuse equals the sum of the squares upon the legs of the right triangle."

The Pythagorean Theorem

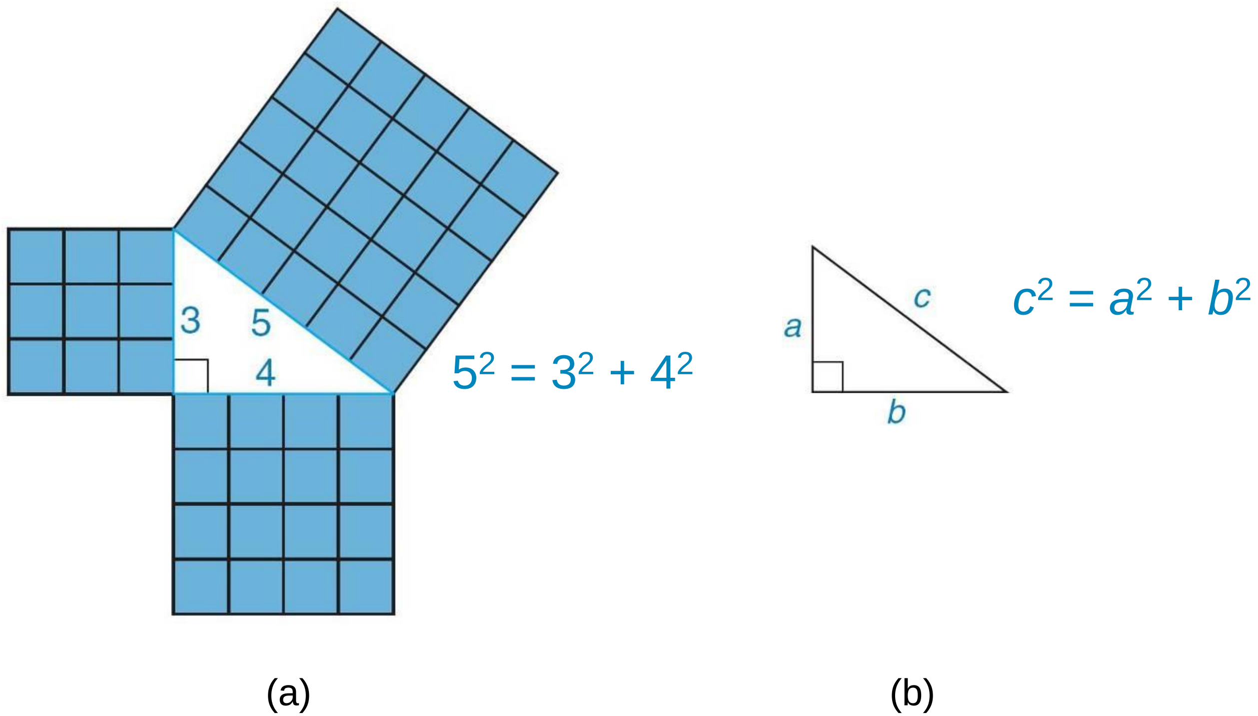


Figure 4.25

See Figure 4.25(a). This interpretation involves the area concept.

The Pythagorean Theorem

By counting squares in Figure 4.25(a), one sees that 25 “square units” is the sum of 9 and 16 square units.

Our interpretation of the Pythagorean Theorem uses number (length) relationships.

The Pythagorean Theorem

When right angle relationships exist in quadrilaterals, we can often apply the “rule of Pythagoras”.

The Pythagorean Theorem In a right triangle with hypotenuse of length c and legs of lengths a and b, it follows that $c^2 = a^2 + b^2$.

Provided that the lengths of two of the sides of a right triangle are known, the Pythagorean Theorem can be applied to determine the length of the third side.

The Pythagorean Theorem

In Example 3, we seek the length of the hypotenuse in a right triangle whose lengths of legs are known.

When we are using the Pythagorean Theorem, c must represent the length of the hypotenuse; however, either leg can be chosen for length a (or b).

Example 3

What is the length of the hypotenuse of a right triangle whose legs measure 6 in. and 8 in.? (See Figure 4.26.)

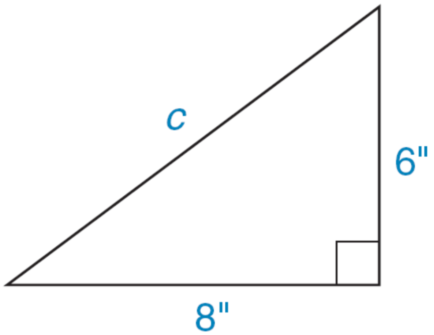


Figure 4.26

Solution:

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 8^2$$

$$c^2 = 36 + 64 \rightarrow c^2 = 100 \rightarrow c = 10 \text{ in.}$$