Chapter 4

Quadrilaterals

4.2 The Parallelogram and Kite

The Parallelogram and Kite

The quadrilaterals discussed in this section have two pairs of congruent sides.

THE PARALLELOGRAM

The Parallelogram

In this section, we find that quadrilaterals with certain characteristics must be parallelograms.

Example 1

Give a formal proof of Theorem 4.2.1.

Theorem 4.2.1

If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Given: In Figure 4.12(a), $\overline{RS} \parallel \overline{VT}$ and $\overline{RS} \cong \overline{VT}$

Prove: *RSTV* is a $\square$

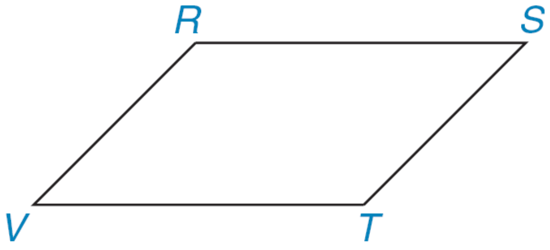

Figure 4.12(a)

Example 1

cont'd

Proof:

Statements	Reasons
1. $\overline{RS} \parallel \overline{VT}$ and $\overline{RS} \cong \overline{VT}$	1. Given
2. Draw diagonal $\overline{VS}$, as in Figure 4.12(b)	2. Exactly one line passes through two points
3. $\overline{VS} \cong \overline{VS}$	3. Identity

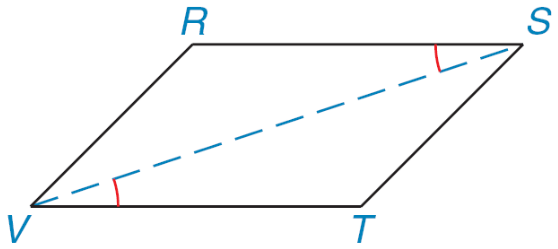

Figure 4.12(b)

Example 1

cont'd

Statements	Reasons
4. $\angle RSV \cong \angle SVT$	4. If two $\parallel$ lines are cut by a transversal, alternate interior $\angle$s are $\cong$
5. $\triangle RSV \cong \triangle TVS$	5. SAS
6. $\angle RVS \cong \angle VST$	6. CPCTC

Example 1

cont'd

Statements	Reasons
7. $\overline{RV} \parallel \overline{ST}$	7. If two lines are cut by a transversal so that alternate interior $\angle$s are $\cong$, these lines are $\parallel$
8. *RSTV* is a ▱	8. If both pairs of opposite sides of a quadrilateral are $\parallel$, the quadrilateral is a parallelogram

The Parallelogram

Theorem 4.2.2

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Another quality of quadrilaterals that determines a parallelogram is stated in Theorem 4.2.3.

Theorem 4.2.3

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

The Parallelogram

When a figure is drawn to represent the hypothesis of a theorem, we should not include more conditions than the hypothesis states.

Relative to Theorem 4.2.3, if we drew two diagonals that not only bisected each other but also were equal in length, then the quadrilateral would be the special type of parallelogram known as a **rectangle**.

THE KITE

The Kite

The next quadrilateral we consider is known as a *kite*, a quadrilateral that gets its name from the child's toy pictured below. In the construction of the kite, there are two pairs of congruent *adjacent* sides. See Figure 4.14(a).

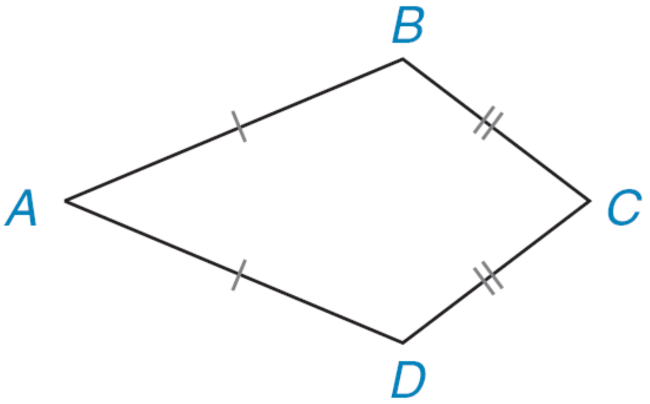

Figure 4.14(a)

The Kite

Definition

A **kite** is a quadrilateral with two distinct pairs of congruent adjacent sides.

The word *distinct* is used in this definition of kite to clarify that the kite does not have four congruent sides.

The Kite

Theorem 4.2.4

In a kite, one pair of opposite angles are congruent.

In Example 2, we verify Theorem 4.2.4 by proving that $\angle B \cong \angle D$. With congruent sides as marked, $\angle A \ncong \angle C$ in kite *ABCD*.

Example 2

Complete the proof of Theorem 4.2.4.

Given: Kite *ABCD* with congruent sides as marked.

[See Figure 4.14(a).]

Prove: $\angle B \cong \angle D$

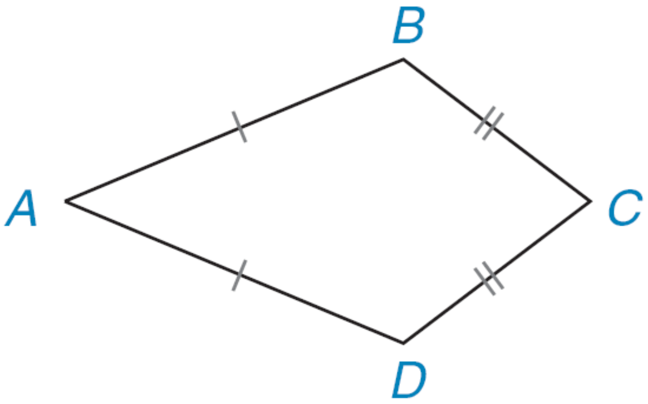

Figure 4.14(a)

Example 2

cont'd

Proof:

Statements	Reasons
1. Kite *ABCD*	1. ?
2. $\overline{BC} \cong \overline{CD}$ and $\overline{AB} \cong \overline{AD}$	2. A kite has two pairs of $\cong$ adjacent sides
3. Draw $\overline{AC}$ [Figure 4.14(b)]	3. Through two points, there is exactly one line

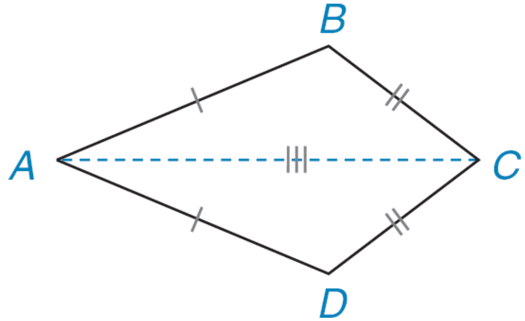

Figure 4.14(b)

Example 2

cont'd

Statements	Reasons
4. $\overline{AC} \cong \overline{AC}$	4. ?
5. $\triangle ACD \cong \triangle ACB$	5. ?
6. ?	6. CPCTC

The Kite

When observing an old barn or shed, we often see that it has begun to lean.

Unlike a triangle, which is rigid in shape [Figure 4.15(a)] and bends only when broken, a quadrilateral [Figure 4.15(b)] does *not* provide the same level of strength and stability.

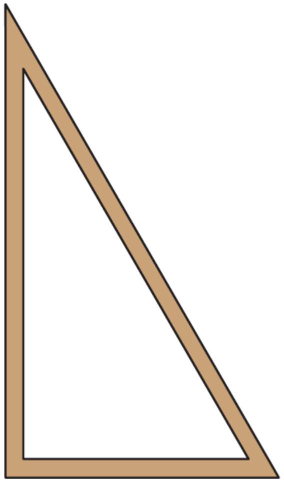

Figure 4.15(a)

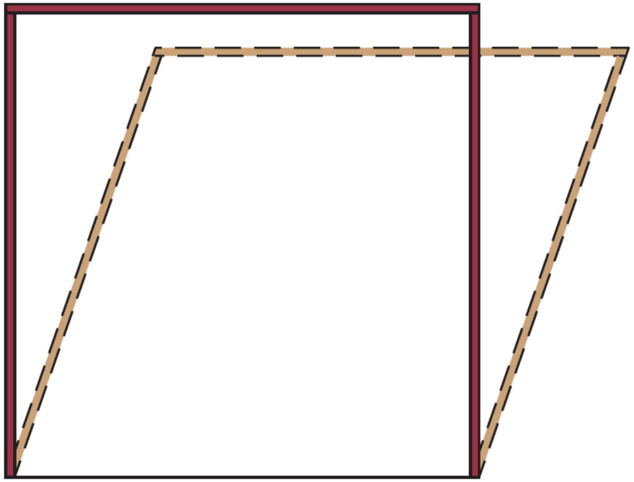

Figure 4.15(b)

The Kite

In the construction of a house, bridge, building, or swing set [Figure 4.15(c)], note the use of wooden or metal triangles as braces.

The brace in the swing set in Figure 4.15(c) suggests the following theorem.

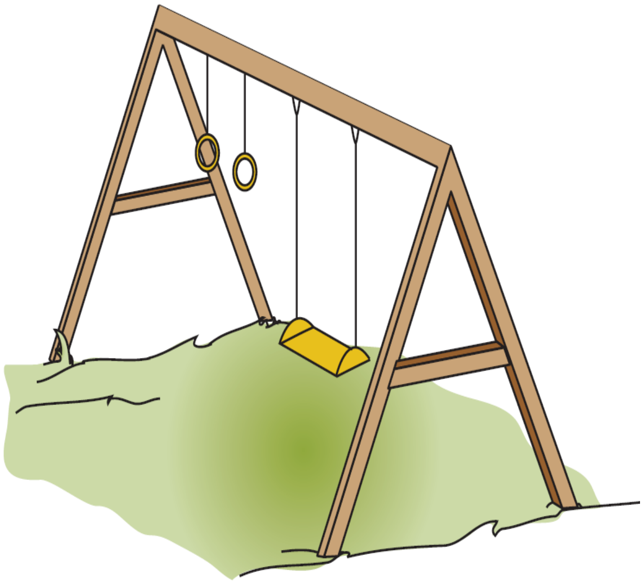

Figure 4.15(c)

The Kite

Theorem 4.2.5

The line segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.

Refer to Figure 4.16(a); in which *M* is the midpoints of $\overline{AB}$, while *N* is the midpoint of $\overline{AC}$. Theorem 4.2.5 claims that $\overline{MN} \parallel \overline{BC}$ and $MN = \frac{1}{2}(BC)$.

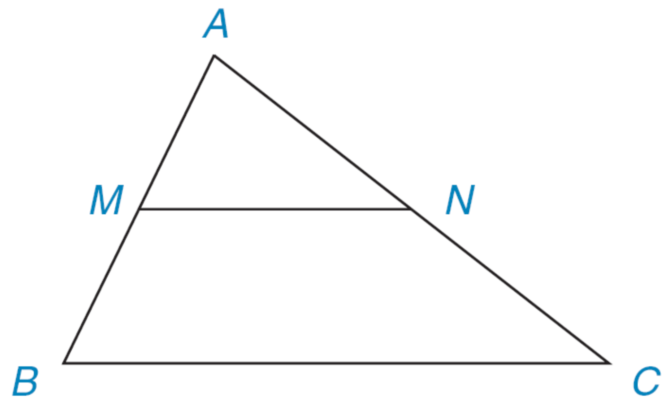

Figure 4.16(a)

Example 3

In $\triangle RST$ in Figure 4.17, M and N are the midpoints of $\overline{RS}$ and $\overline{RT}$, respectively.

a) If $ST = 12.7$, find MN.

b) If $MN = 15.8$, find ST.

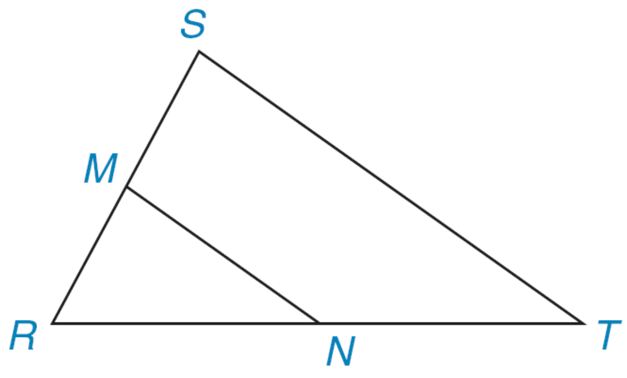

Figure 4.17

Example 3 – *Solution*

a) $MN = \frac{1}{2}(ST)$, so $MN = \frac{1}{2}(12.7)$

$= 6.35.$

b) $MN = \frac{1}{2}(ST)$, so $15.8 = \frac{1}{2}(ST)$.

Multiplying by 2, we find that $ST = 31.6$.