Chapter 4

Quadrilaterals

4.1 Properties of a Parallelogram

Properties of a Parallelogram

A **quadrilateral** is a polygon that has exactly four sides.

Unless otherwise stated, the term *quadrilateral* refers to a plane figure such as *ABCD* in Figure 4.1(a), in which the line segment sides lie within a single plane.

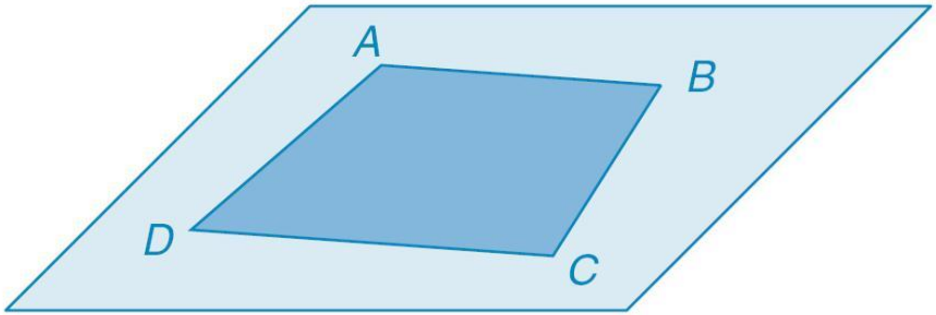


Figure 4.1(a)

Properties of a Parallelogram

When two sides of the quadrilateral are skew (not coplanar), as with *MNPQ* in Figure 4.1(b), that quadrilateral is said to be **skew.**

Thus, *MNPQ* is a skew quadrilateral. We generally consider quadrilaterals whose sides are coplanar.

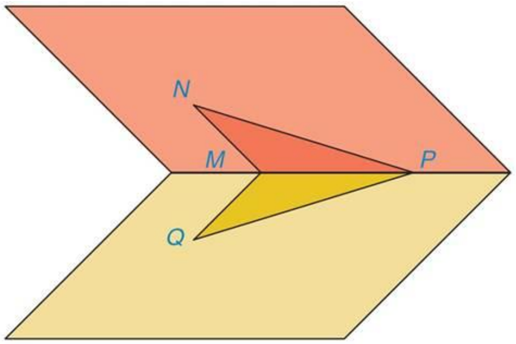


Figure 4.1(b)

Properties of a Parallelogram

Definition

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

Because the symbol for parallelogram is ▱, the quadrilateral in Figure 4.2(b) is ▱ *RSTV.*

The set is P = {parallelograms} is a subset of Q = {quadrilaterals}.

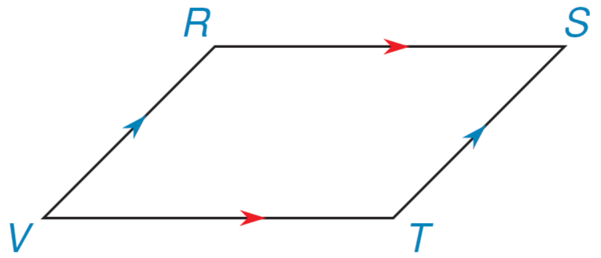


Figure 4.2 (b)

Example 1

Theorem 4.1.1

A diagonal of a parallelogram separates it into two congruent triangles.

Given: $\square ABCD$ with diagonal $\overline{AC}$ (See Figure 4.3.)

Prove: $\triangle ACD \cong \triangle CAB$

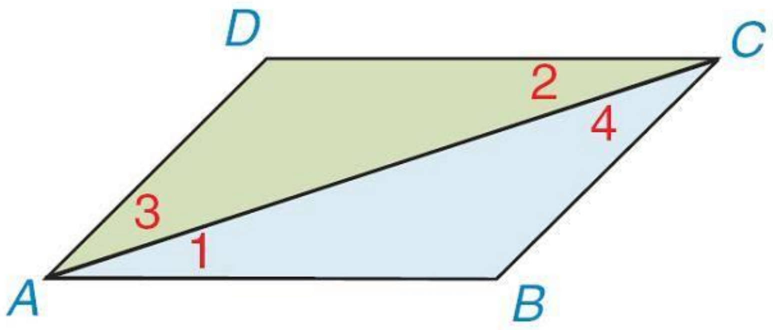


Figure 4.3

Example 1

cont'd

Proof:

Statements	**Reasons**
1. ▱$ABCD$	1. Given
2. $\overline{AB} \parallel \overline{CD}$	2. The opposite sides of a ▱ are $\parallel$ (definition)
3. $\angle 1 \cong \angle 2$	3. If two $\parallel$ lines are cut by a transversal, the alternate interior $\angle$s are congruent

Example 1

cont'd

Statements	Reasons
4. $\overline{AD} \parallel \overline{BC}$	4. Same as reason 2
5. $\angle 3 \cong \angle 4$	5. Same as reason 3
6. $\overline{AC} \cong \overline{AC}$	6. Identity
7. $\triangle ACD \cong \triangle CAB$	7. ASA

Properties of a Parallelogram

Corollary 4.1.2

The opposite angles of a parallelogram are congruent.

Corollary 4.1.3

The opposite sides of a parallelogram are congruent.

Corollary 4.1.4

The diagonals of a parallelogram bisect each other.

Properties of a Parallelogram

Recall Theorem 2.1.4: “If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.”

A corollary of that theorem is stated next.

Corollary 4.1.5

Two consecutive angles of a parallelogram are supplementary.

Theorem 4.1.6

Two parallel lines are everywhere equidistant.

Properties of a Parallelogram

Definition

An **altitude** of a parallelogram is a line segment drawn from one side so that it is perpendicular to the nonadjacent side (or to an extension of that side).

For ▱*RSTV*, $\overline{RW}$ and $\overline{SX}$ are altitudes to side $\overline{VT}$ (or to side $\overline{RS}$), as shown in Figure 4.5(a).

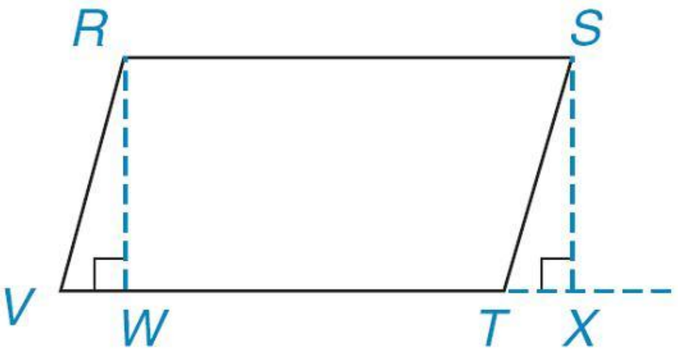


Figure 4.5(a)

With respect to side $\overline{RS}$, sometimes called base $\overline{RS}$, the length *RW* (or *SX*) is the *height* of *RSTV*.

Properties of a Parallelogram

Similarly, in Figure 4.5(b), $\overline{TY}$ and $\overline{SZ}$ are altitudes to side $\overline{RV}$ (or to side $\overline{ST}$). Also, the length *TY* (or *ZS*) is called the *height* of parallelogram *RSTV* with respect to side $\overline{ST}$ (or $\overline{RV}$).

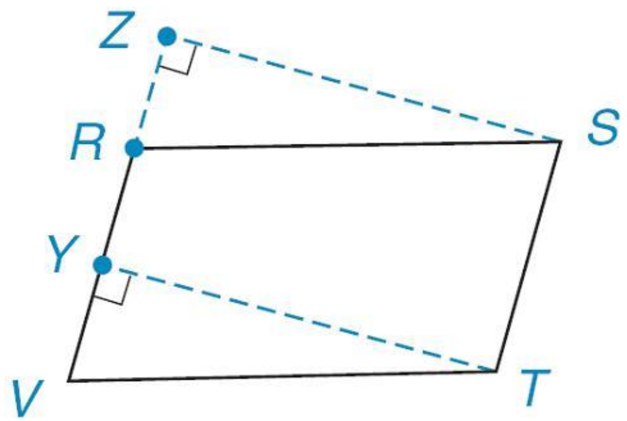


Figure 4.5(b)

Properties of a Parallelogram

Next we consider an inequality relationship for the parallelogram.

To develop this relationship, we need to investigate an inequality involving two triangles.

Properties of a Parallelogram

Lemma 4.1.7

If two sides of one triangle are congruent to two sides of a second triangle and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.

Properties of a Parallelogram

Given: $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$; m∠$B$ > m∠E
(See Figure 4.6.)

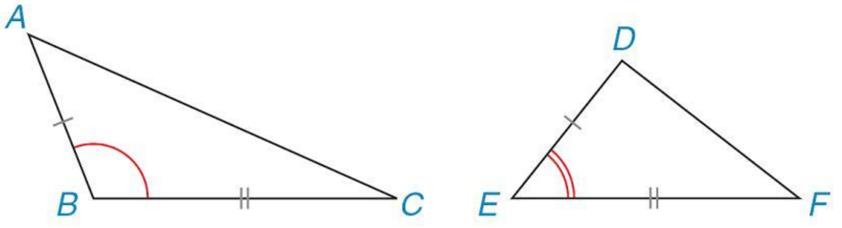


Figure 4.6

Prove: $AC > DF$

Properties of a Parallelogram

Now we can compare the lengths of the diagonals of a parallelogram.

For a parallelogram having no right angles, two consecutive angles are unequal but supplementary; thus, one angle of the parallelogram will be acute and the consecutive angle will be obtuse.

Properties of a Parallelogram

In Figure 4.7(a), ▱*ABCD* has acute angle *A* and obtuse angle *D*.

Note that the two sides of the triangles that include $\angle A$ and $\angle D$ are congruent.

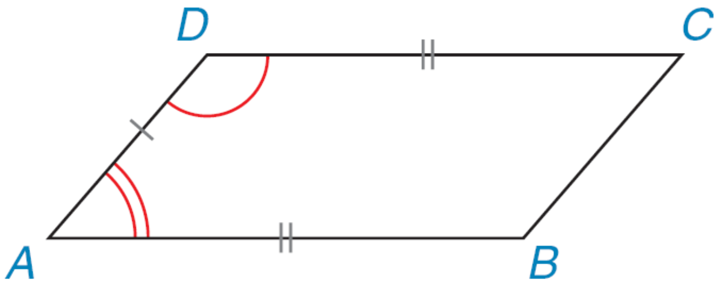


Figure 4.7(a)

Properties of a Parallelogram

In Figure 4.7(b), diagonal $\overline{AC}$ lies opposite the obtuse angle *ADC* in $\triangle ACD$, and diagonal $\overline{BD}$ lies opposite the acute angle *DAB* in $\triangle ABD$.

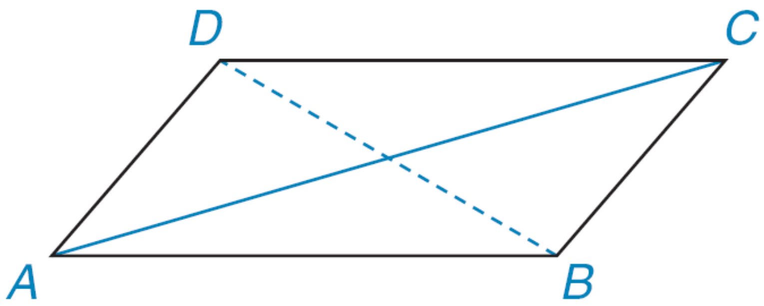
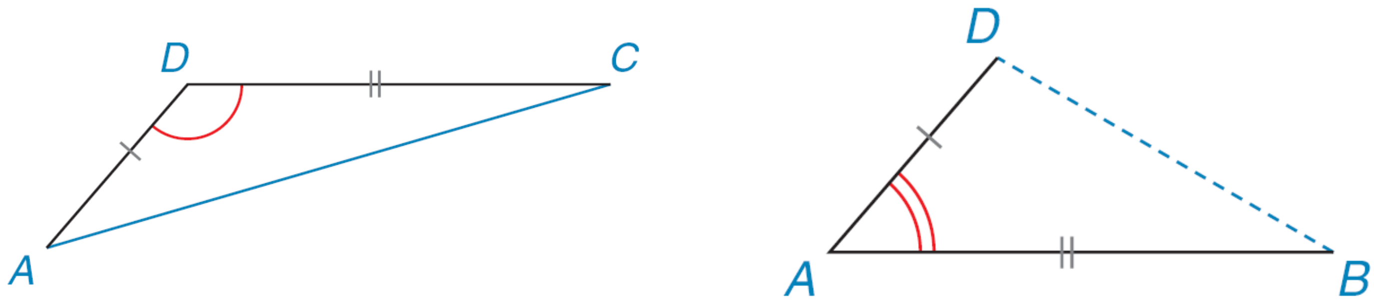


Figure 4.7(b)

Properties of a Parallelogram

In Figures 4.7(c) and (d), we have taken $\triangle ACD$ and $\triangle ABD$ from $\square ABCD$ of Figure 4.7(b).

Note that $\overline{AC}$ (opposite obtuse $\angle D$) is longer than $\overline{DB}$ (opposite acute $\angle A$).



(c) (d)

Figure 4.7

Properties of a Parallelogram

On the basis of Lemma 4.1.7 and the preceding discussion, we have the following theorem.

Theorem 4.1.8

In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.

SPEED AND DIRECTION OF AIRCRAFT

Speed and Direction of Aircraft

For the application to follow, we will indicate the velocity of an airplane or of the wind by drawing a directed arrow.

In each case, a scale is used on a grid in which a north-south line meets an east-west line at right angles.

Speed and Direction of Aircraft

Consider the sketches in Figure 4.9 and read their descriptions.

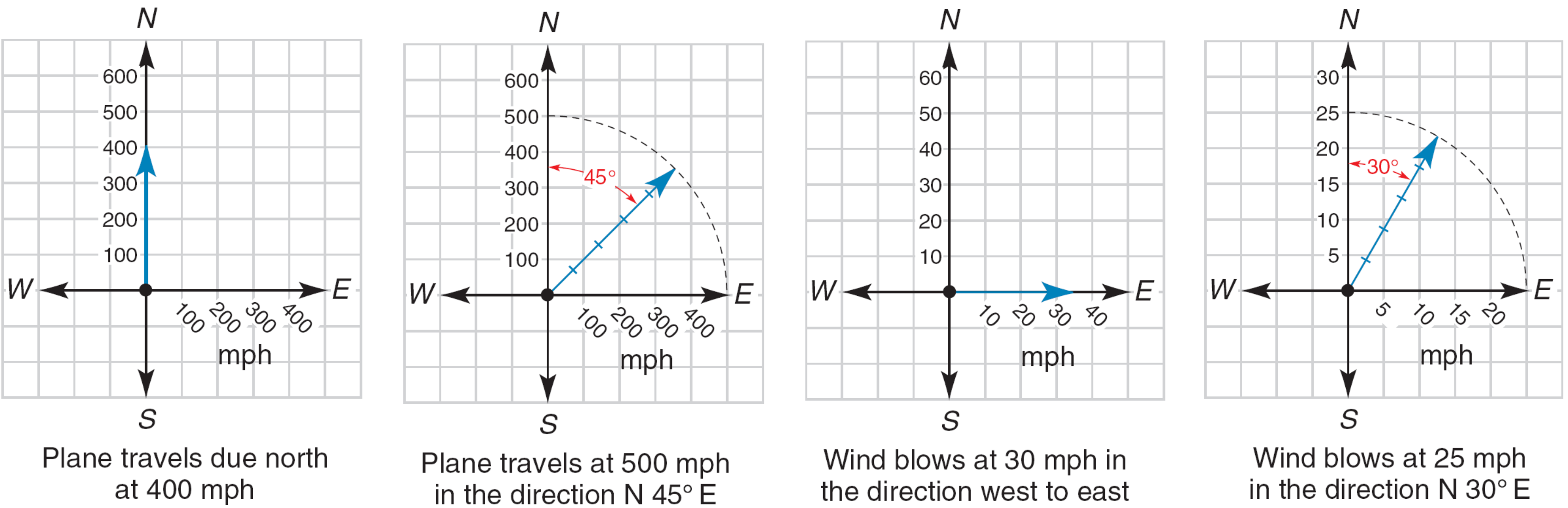


Figure 4.9

Speed and Direction of Aircraft

In some scientific applications, a parallelogram can be used to determine the solution to the problem.

For instance, the Parallelogram Law enables us to determine the resulting speed and direction of an airplane when the velocity of the airplane and that of the wind are considered together.

Speed and Direction of Aircraft

In Figure 4.10, the arrows representing the two velocities are placed head-to-tail from the point of origin. Because the order of the two velocities is reversible, the drawing leads to a parallelogram.

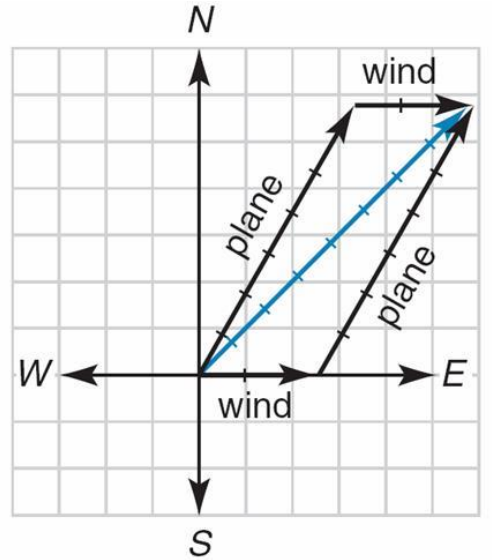


Figure 4.10

Speed and Direction of Aircraft

In the parallelogram, it is the length and direction of the indicated diagonal that solve the problem.

Accuracy is critical in scaling the drawing that represents the problem. Otherwise, the ruler and protractor will give poor results in your answer.

Example 7

An airplane travels due north at 500 kph. If the wind blows at 50 kph from west to east, what are the resulting speed and direction of the plane?

Solution:

Using a ruler to measure the diagonal of the parallelogram, we find that the length corresponds to a speed of approximately 505 kph.

Using a protractor, we find that the direction is approximately N 6° E. (See Figure 4.11.)

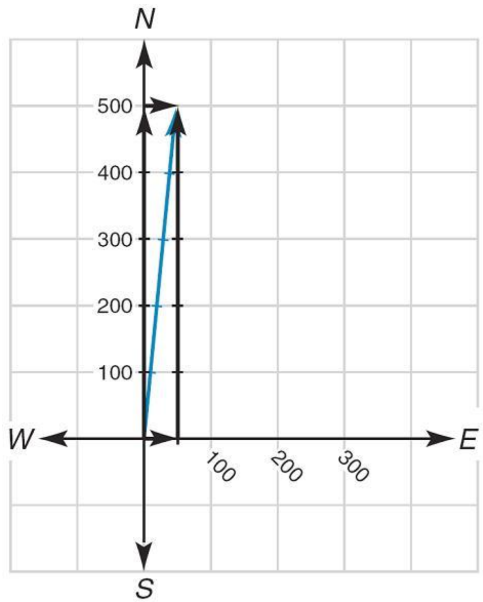


Figure 4.11