Chapter 3

# Triangles

# 3.2 Corresponding Parts of Congruent Triangles

# Corresponding Parts of Congruent Triangles

Recall that the definition of congruent triangles states that *all* six parts (three sides and three angles) of one triangle are congruent respectively to the six corresponding parts of the second triangle.

If we have proved that $\Delta ABC \cong \Delta DEF$ by SAS (the congruent parts are marked in Figure 3.14), then we can draw conclusions such as $\angle C \cong \angle F$ and $\overline{AC} \cong \overline{DF}$.

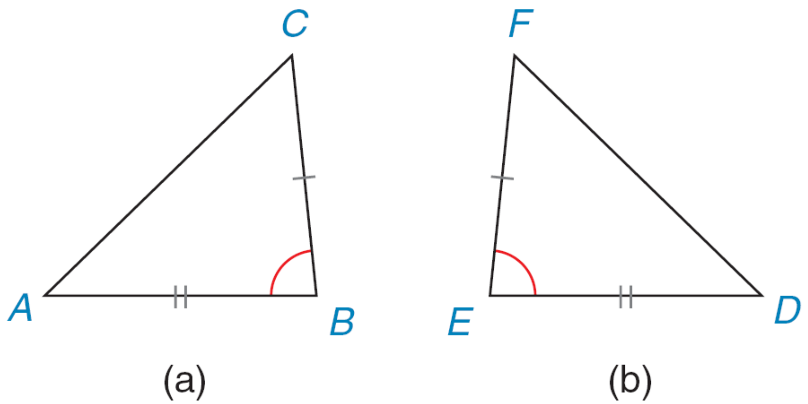



**Figure 3.14**

# Corresponding Parts of Congruent Triangles

The following reason (CPCTC) is often cited for drawing such conclusions and is based on the definition of congruent triangles.

**CPCTC**

Corresponding parts of congruent triangles are congruent.

# Example 1

Given: $\overrightarrow{WZ}$ bisects $\angle TWV$

$$\overline{WT} \cong \overline{WV}$$

(See Figure 3.15.)

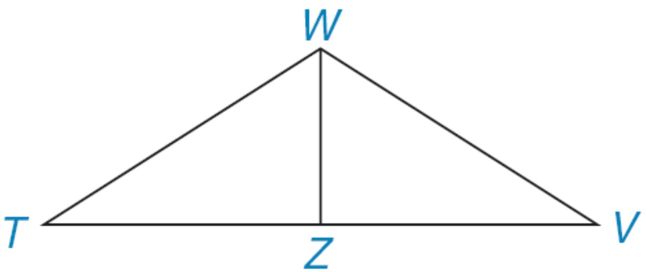


**Figure 3.15**

Prove: $\overline{TZ} \cong \overline{VZ}$

# Example 1
cont'd

Proof:

| Statements | Reasons |
| --- | --- |
| 1. $\overrightarrow{WZ}$ bisects $\angle TWV$ | 1. Given |
| 2. $\angle TWZ \cong \angle VWZ$ | 2. The bisector of an angle separates it into two $\cong \angle$s |
| 3. $\overline{WT} \cong \overline{WV}$ | 3. Given |
| 4. $\overline{WZ} \cong \overline{WZ}$ | 4. Identity |
| 5. $\triangle TWZ \cong \triangle VWZ$ | 5. SAS |
| 6. $\overline{TZ} \cong \overline{VZ}$ | 6. CPCTC |

# Corresponding Parts of Congruent Triangles

In Example 1, we could just as easily have used CPCTC to prove that two angles are congruent.

If we had been asked to prove that $\angle T \cong \angle V$, then the final statement of the proof would have read

| | |
|---|---|
| 6. $\angle T \cong \angle V$ | 6. CPCTC |

# Corresponding Parts of Congruent Triangles

We can take the proof in Example 1 a step further by proving triangles congruent, using CPCTC, and finally reaching another conclusion such as parallel or perpendicular lines.

In Example 1, suppose we had been asked to prove that $\overline{WZ}$ bisects $\overline{TV}$.

Then Steps 1–6 of Example 1 would have remained as they are, and a seventh step of the proof would have read

| | |
|---|---|
| 7. $\overline{WZ}$ bisects $\overline{TV}$ | 7. If a line segment is divided into two $\cong$ parts, then it has been bisected |

# RIGHT TRIANGLES

# Right Triangles

In a right triangle, the side opposite the right angle is the **hypotenuse** of that triangle, and the sides of the right angle are the **legs** of the triangle. These parts of a right triangle are illustrated in Figure 3.18.

Another method for proving triangles congruent is the HL method, which applies exclusively to right triangles.

In HL, H refers to hypotenuse and L refers to leg.

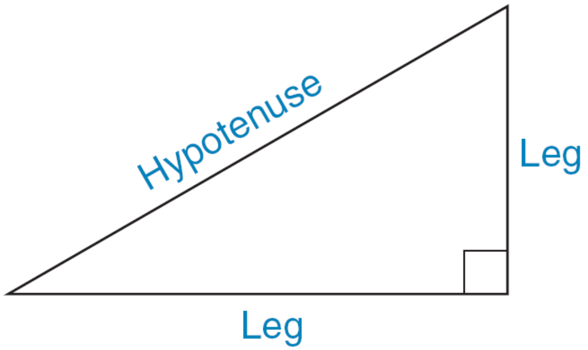



**Figure 3.18**

# HL (METHOD FOR PROVING TRIANGLES CONGRUENT)

# HL (Method for Proving Triangles Congruent)

Theorem 3.2.1

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent (HL).

The relationships described in Theorem 3.2.1 (HL) are illustrated in Figure 3.19. In Example 3, the construction based upon HL leads to a unique right triangle.

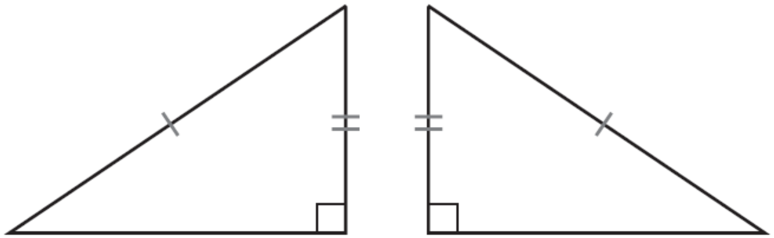


**Figure 3.19**

# Example 3

Given:

$\overline{AB}$ and $\overline{CA}$ in Figure 3.20(a); note that $AB > CA$.

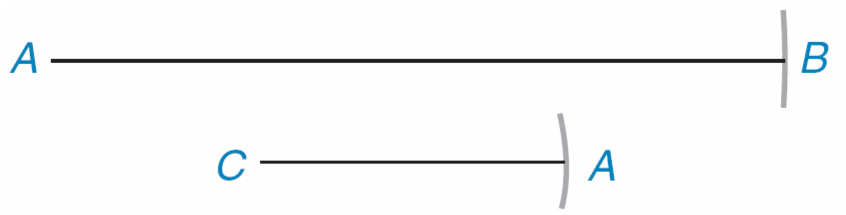



**Figure 3.20(a)**

Construct:

The right triangle with hypotenuse of length equal to $AB$ and one leg of length equal to $CA$

# Example 3 – *Solution*

Figure 3.20(b): Construct $\overleftrightarrow{CQ}$ perpendicular to $\overleftrightarrow{EF}$ at point *C*.

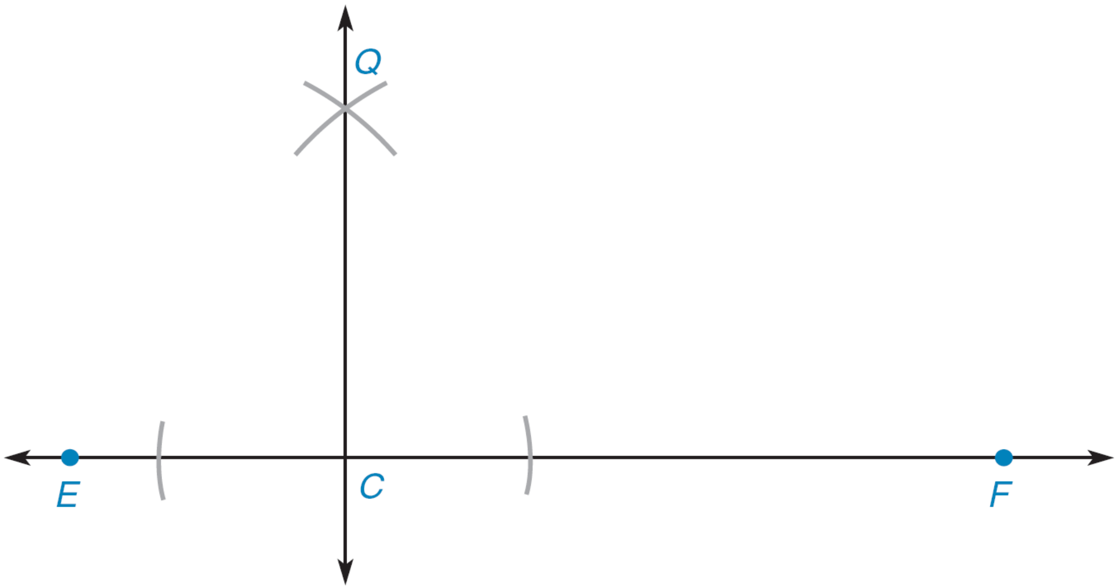



**Figure 3.20(b)**

# Example 3 – *Solution*

cont'd

Figure 3.20(c): Now mark off the length of $\overline{CA}$ on $\overleftrightarrow{CQ}$ .

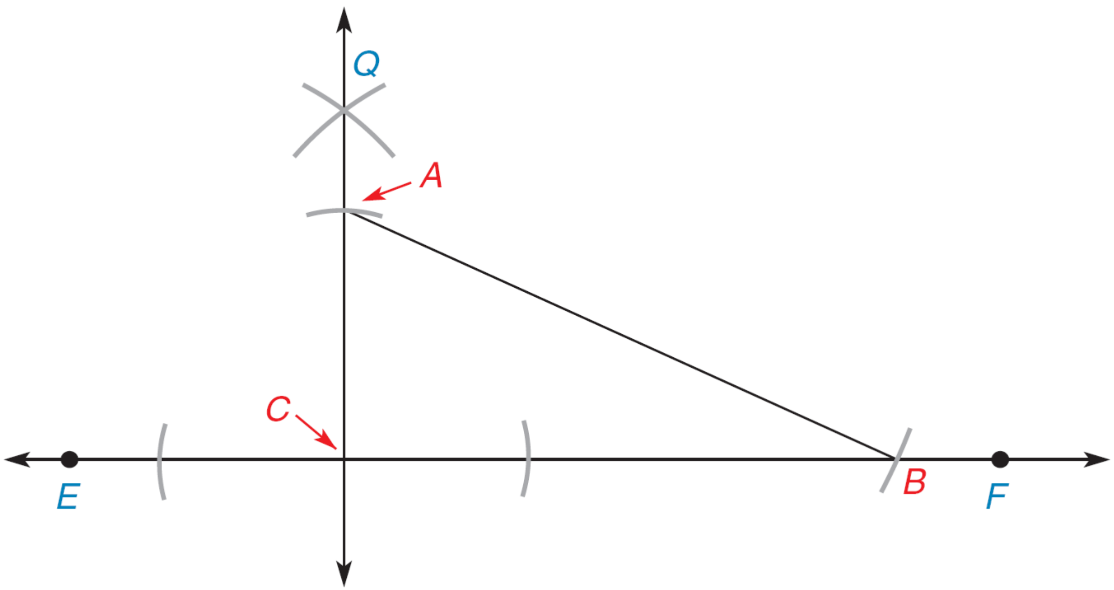



**Figure 3.20(c)**

Finally, with point *A* as center, mark off an arc with its length equal to that of $\overline{AB}$ as shown. $\triangle ABC$ is the desired right $\triangle$.

# THE PYTHAGOREAN THEOREM

# The Pythagorean Theorem

The following theorem can be applied only when a triangle is a right triangle.

## Pythagorean Theorem

The square of the length ($c$) of the hypotenuse of a right triangle equals the sum of squares of the lengths ($a$ and $b$) of the legs of the right triangle; that is, $c^2 = a^2 + b^2$.

In applications of the Pythagorean Theorem, we often find statements such as $c^2 = 25$.

Using the following property, we see that $c = \sqrt{25}$ or $c = 5$.

# The Pythagorean Theorem

## Square Roots Property

Let $x$ represent the length of a line segment, and let $p$ represent a positive number. If $x^2 = p$, then $x = \sqrt{p}$.

The *square root* of $p$, symbolized $\sqrt{p}$, represents the number that when multiplied times itself equals $p$.

As we indicated earlier, $\sqrt{25} = 5$ because $5\sqrt{25} = 25$.

# The Pythagorean Theorem

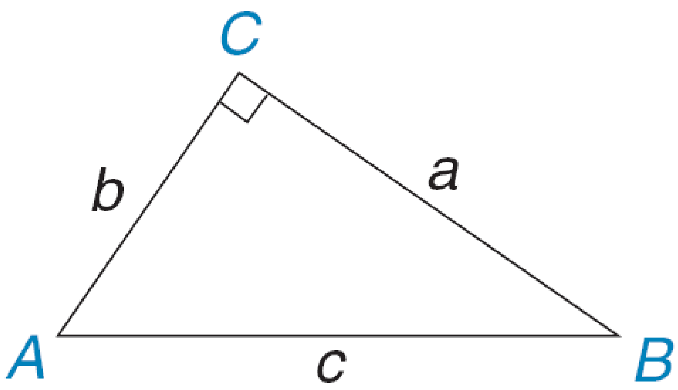
When a square root is not exact, a calculator can be used to find its approximate value; where the symbol $\approx$ means "is equal to approximately," $\sqrt{22} \approx 4.69$ because $4.69 \sqrt{4.69} = 21.9961 \approx 22$.

# Example 6

Find the length of the third side of the right triangle. See the figure below.

a) Find $c$ if $a = 6$ and $b = 8$.

b) Find $b$ if $a = 7$ and $c = 10$.



# Example 6(a) – *Solution*

$$c^2 = a^2 + b^2,$$

so $\quad c^2 = 6^2 + 8^2$

or $\quad c^2 = 36 + 64$

$$= 100.$$

Then $c = \sqrt{100}$

$$= 10.$$

# Example 6(b) – *Solution*

cont'd

$$c^2 = a^2 + b^2,$$

so $\quad 10^2 = 7^2 + b^2$

or $\quad 100 = 49 + b^2$

Subtracting yields

$$b^2 = 51,$$

so $\quad b = \sqrt{51}$

$$\approx 7.14.$$