Chapter 2

Parallel Lines

2.6 Symmetry and Transformations

LINE SYMMETRY

Line Symmetry

In the figure below, rectangle *ABCD* is said to have *symmetry with respect to line* ℓ because each point to the left of the *line of symmetry* or *axis of symmetry* has a corresponding point to the right; for instance, *X* and *Y* are *corresponding points.*

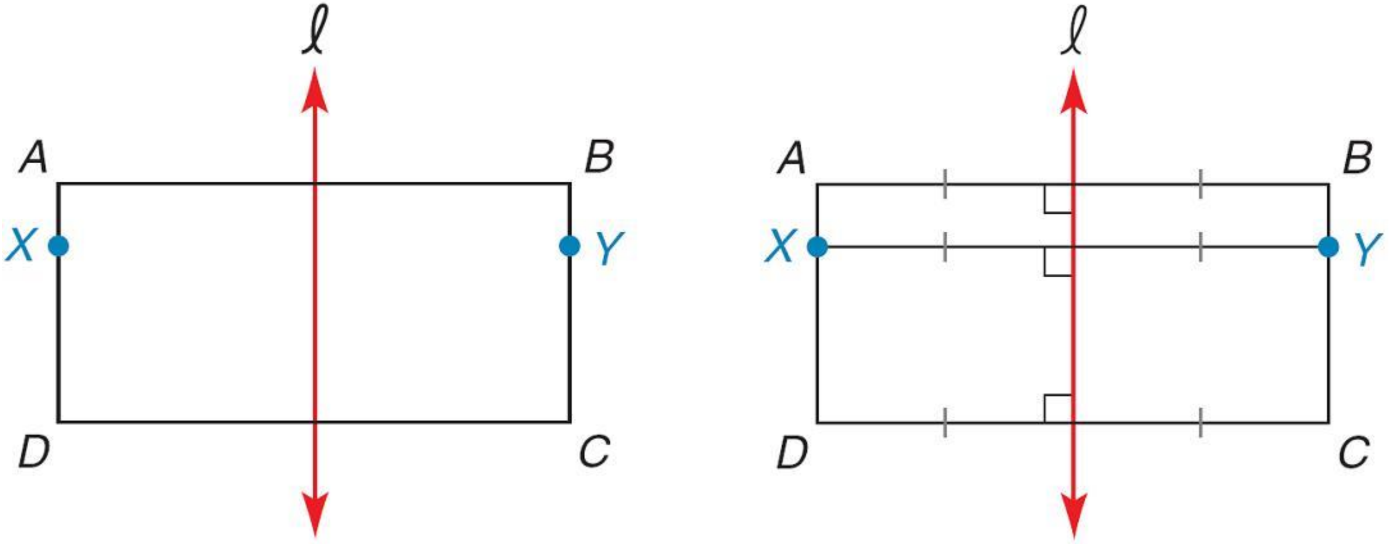


Figure 2.38

Line Symmetry

Definition

A figure has *symmetry with respect to a line* ℓ if for every point X on the figure, there is a second point Y on the figure for which ℓ is the perpendicular bisector of $\overline{XY}$.

In particular, *ABCD* of Figure 2.38 has *horizontal symmetry* with respect to line ℓ; that is, a vertical axis of symmetry leads to a pairing of corresponding points on a horizontal line.

In Example 1, we see that a horizontal axis leads to *vertical symmetry* for points on ▭ *ABCD*.

Example 1

Rectangle *ABCD* in Figure 2.38 has a second line of symmetry. Draw this horizontal line (or axis) for which there is *vertical symmetry*.

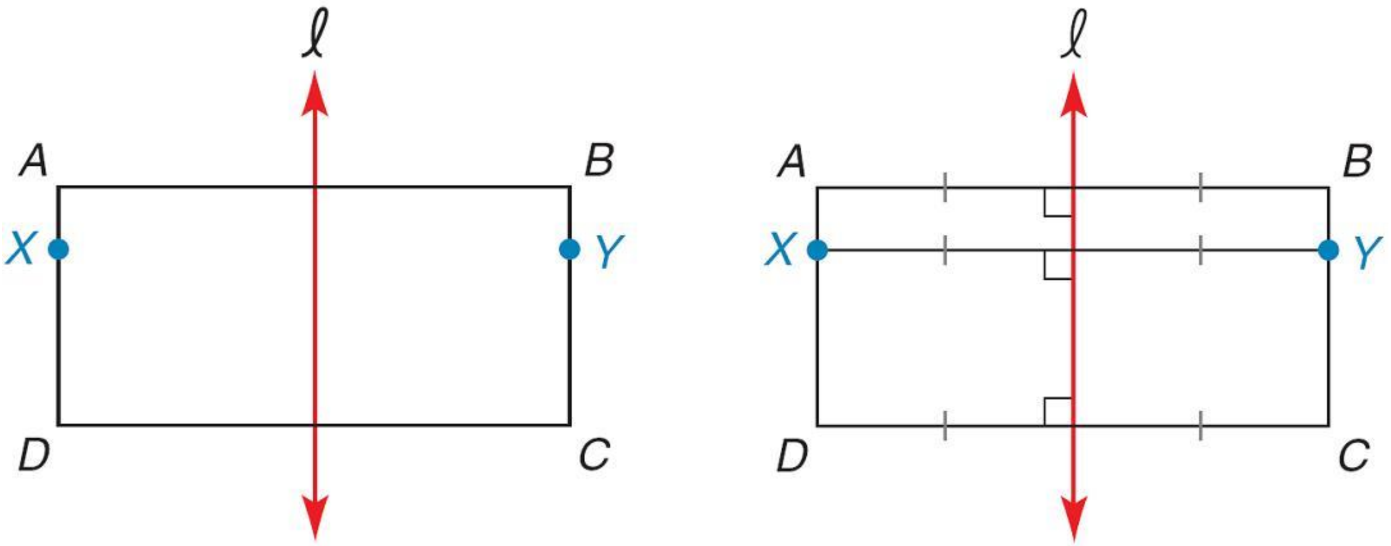
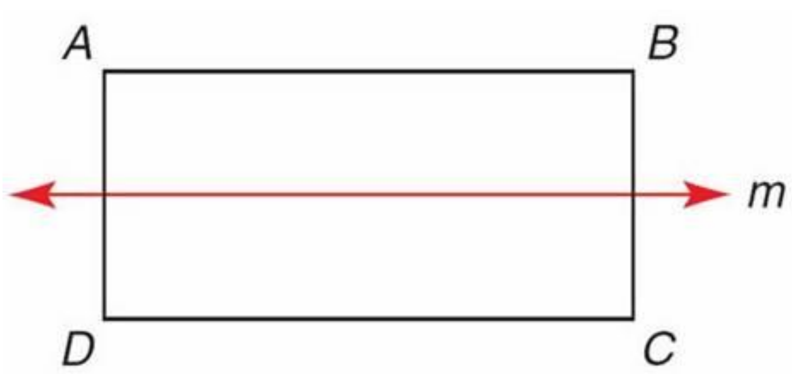


Figure 2.38

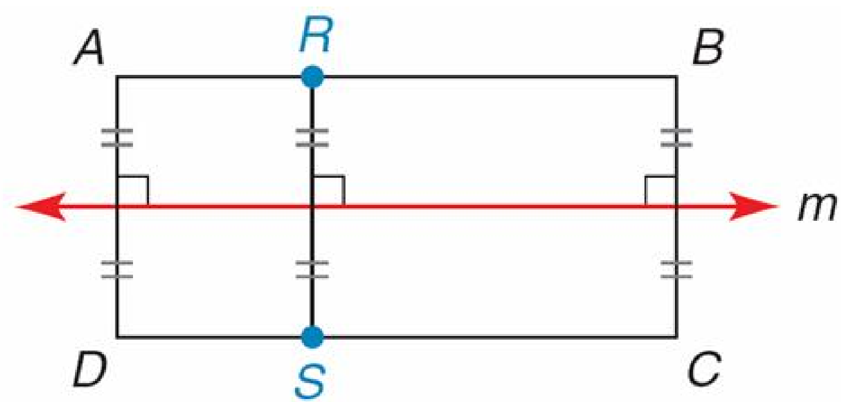
Example 1 – *Solution*

Line *m* (determined by the midpoints of $\overline{AD}$ and $\overline{BC}$) is the desired line of symmetry.

As shown in Figure 2.39(b), *R* and *S* are located symmetrically with respect to line *m*.



(a)



(b)

Figure 2.39

POINT SYMMETRY

Point Symmetry

In Figure 2.41, rectangle *ABCD* is also said to have *symmetry with respect to a point.* As shown, point *P* is determined by the intersection of the diagonals of ▭*ABCD.*

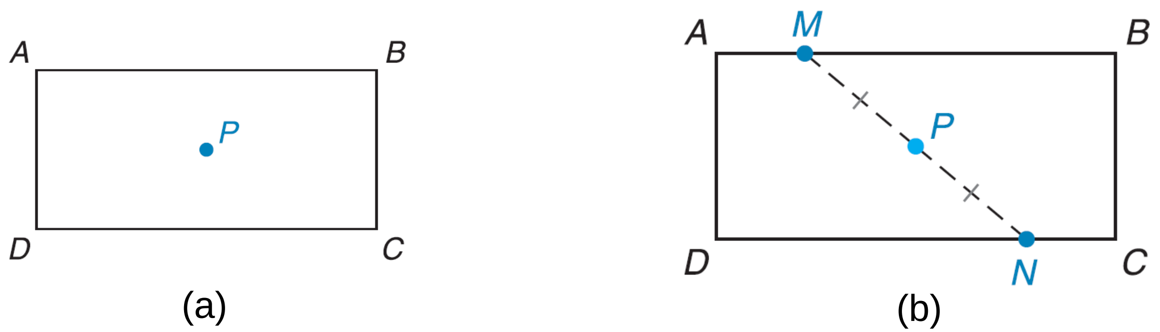


Figure 2.41

Definition

A figure has **symmetry with respect to point *P*** if for every point *M* on the figure, there is a second point *N* on the figure for which point *P* is the midpoint of $\overline{MN}$.

Point Symmetry

On the basis of this definition, each point on ▱*ABCD* in Figure 2.41(a) has a corresponding point that is the same distance from *P* but which lies in the opposite direction from *P.*

In Figure 2.41(b), *M* and *N* are a pair of corresponding points. Even though a figure may have multiple lines of symmetry, a figure can have only one point of symmetry.

Thus, the point of symmetry (when one exists) is unique.

Example 4

Which letter(s) shown below have point symmetry?

M N P S X

Solution:

N, S, and X as shown all have point symmetry.

TRANSFORMATIONS

Transformations

In the following material, we will generate new figures from old figures by association of points.

In particular, the *transformations* included in this textbook will preserve the shape and size of the given figure; in other words, these transformations preserve lengths and angle measures and thus lead to a second figure that is *congruent* to the given figure.

The types of transformations included are

(1) the *slide* or *translation,*

(2) the *reflection,* and

(3) the *rotation*.

Transformations

- **Slides (Translations)**

With this type of transformation, every point of the original figure is associated with a second point by locating it through a movement of a fixed length and direction.

In Figure 2.43, $\triangle ABC$ is translated to the second triangle (its image $\triangle DEF$) by sliding each point through the distance and in the direction that takes point *A* to point *D*.

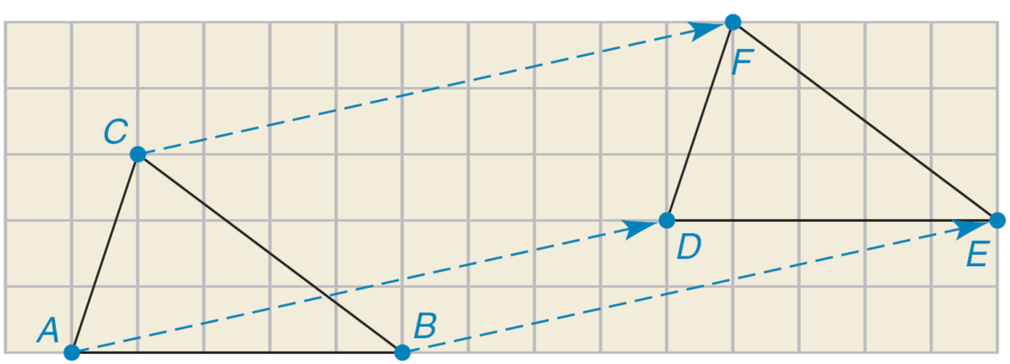


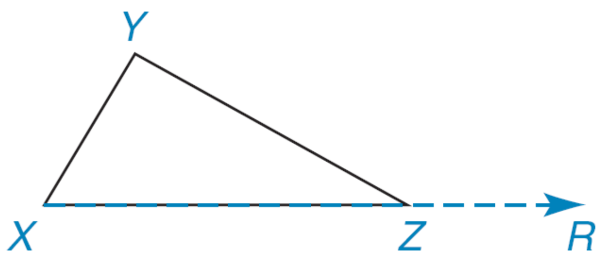
Figure 2.43

Transformations

The background grid is not necessary to demonstrate the slide, but it lends credibility to our claim that the same length and direction have been used to locate each point.

Example 6

Slide $\triangle XYZ$ horizontally in Figure 2.44 to form $\triangle RST$. In this example, the distance (length of the slide) is XR.



Solution:

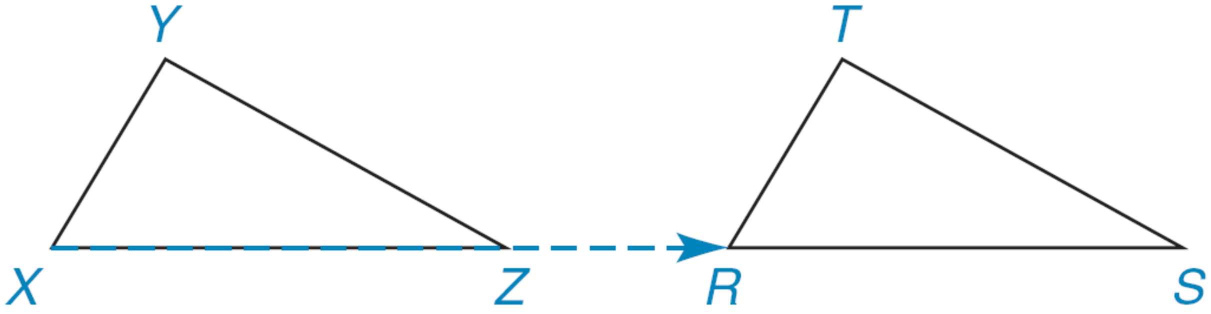


Figure 2.44

Transformations

In Example 6, $\triangle XYZ \cong \triangle RTS$. In every slide, the given figure and the produced figure (its *image*) are necessarily congruent. In Example 6, the correspondence of vertices is given by $X \leftrightarrow R$, $Y \leftrightarrow T$, and $Z \leftrightarrow S$.

Reflections

With the reflection, every point of the original figure is reflected across a line in such a way as to make the given line a line of symmetry.

Each pair of corresponding points will lie on opposite sides of the line of reflection and at like distances.

Transformations

In Figure 2.46, obtuse triangle *MNP* is reflected across the vertical line $\overleftrightarrow{AB}$ to produce the image $\triangle GHK$.

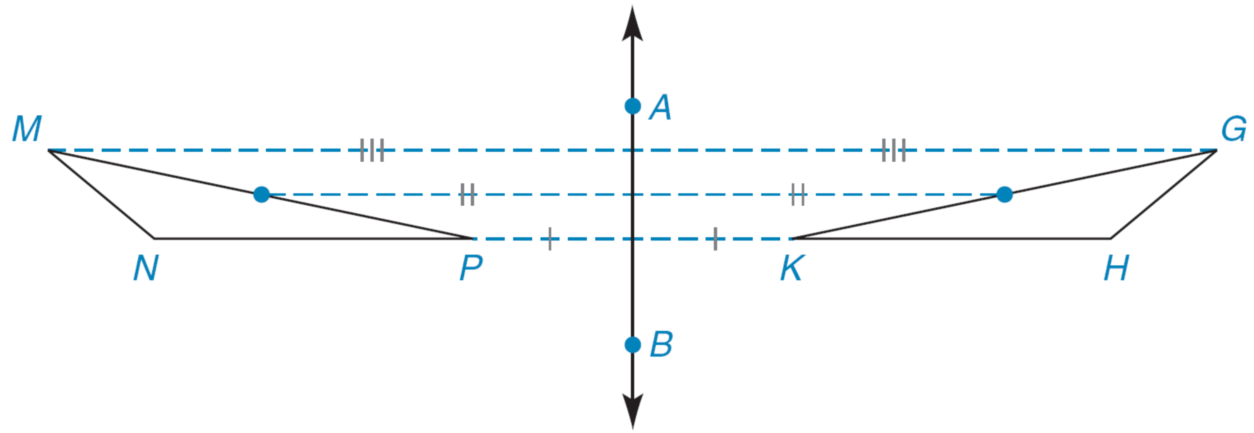


Figure 2.46

The vertex *N* of the given obtuse angle corresponds to the vertex *H* of the obtuse angle in the image triangle.

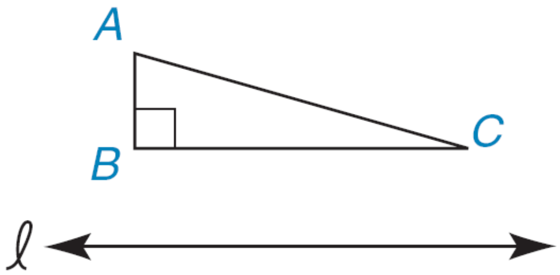
Transformations

With the vertical line as the axis of reflection, a drawing such as Figure 2.46 is sometimes called a *horizontal reflection*, since the image lies to the right of the given figure. It is possible for the line of reflection to be horizontal or oblique (slanted).

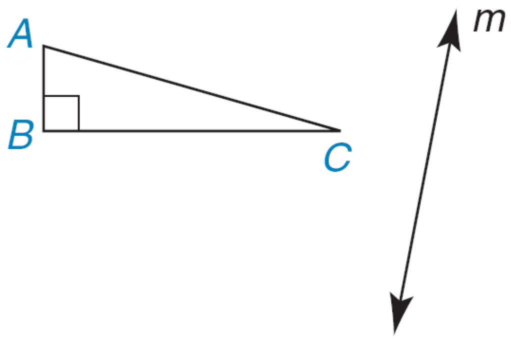
Example 8

Draw the reflection of right $\triangle ABC$

a) across line ℓ to form $\triangle XYZ$.



b) across line *m* to form $\triangle PQR$.



Example 8 – *Solution*

As shown in Figure 2.47

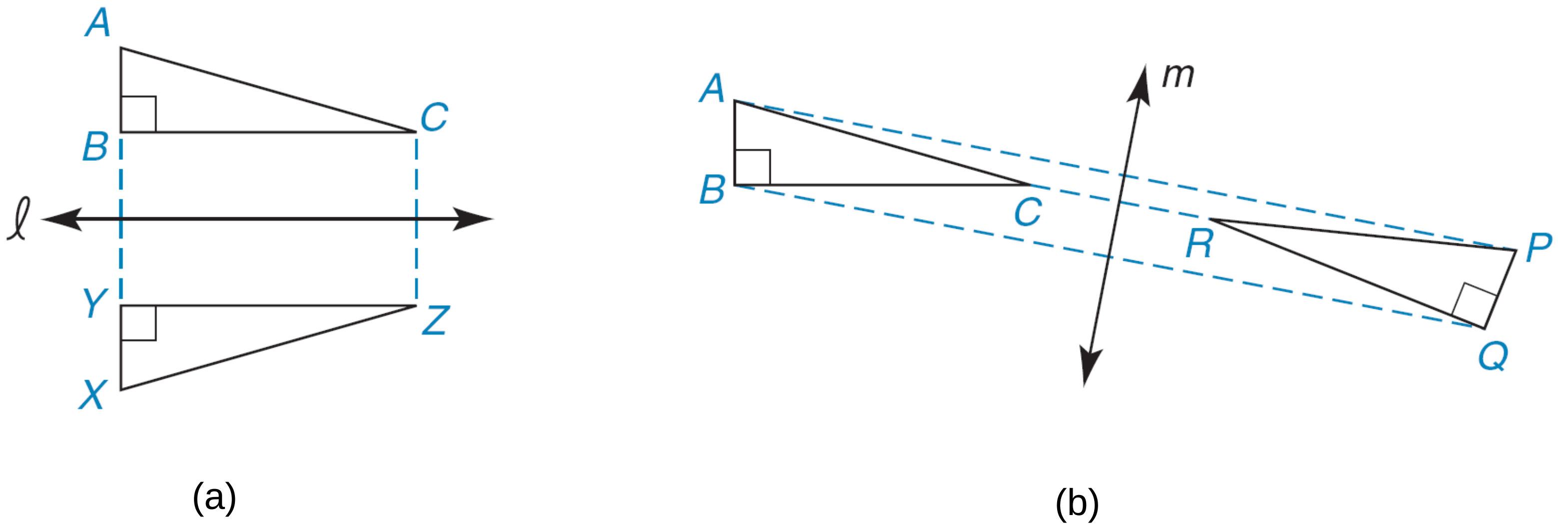


Figure 2.47

Transformations

Rotations

In this transformation, every point of the given figure leads to a point (its image) by rotation about a given point through a prescribed angle measure.

In Figure 2.50, ray *AB* rotates about point *A* clockwise through an angle of 30° to produce the image ray *AC*.

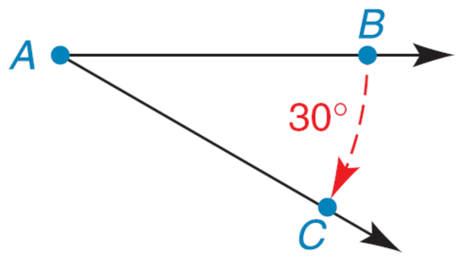


Figure 2.50

This has the same appearance as the second hand of a clock over a five-second period of time. In this figure, $A \leftrightarrow A$ and $B \leftrightarrow C$.

Transformations

As shown in Figure 2.50, the *angle of rotation* measures 30°; also the *center of rotation* is point *A*.

Example 11

In Figure 2.51, square *WXYZ* has been rotated counterclockwise about its center (intersection of diagonals) through an angle of 45° to form congruent square *QMNP*. What is the name of the eight-pointed geometric figure that is formed by the two intersecting squares?

Example 11 – *Solution*

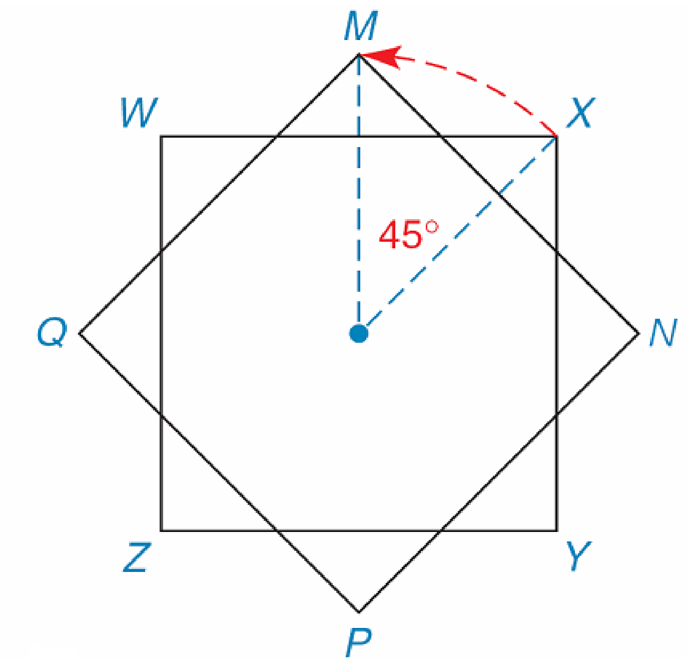


Figure 2.51

The eight-pointed figure formed is a regular octagram.