# Chapter 2 Parallel Lines

# 2.4 The Angles of a Triangle

# The Angles of a Triangle

Definition

A **triangle** (symbol Δ) is the union of three line segments that are determined by three noncollinear points.

The triangle in Figure 2.24 is known as Δ*ABC*, or Δ*BCA*, etc. (any order of letters *A, B,* and *C* can be used).

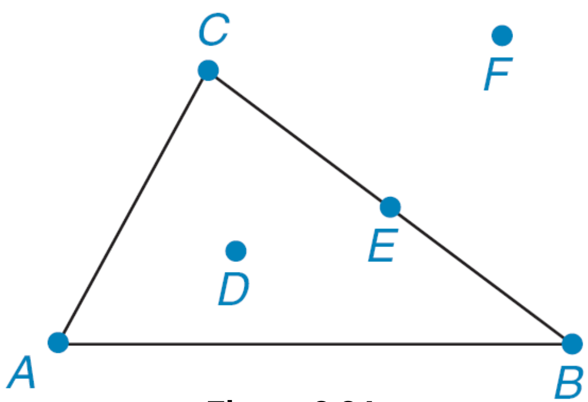



**Figure 2.24**

# The Angles of a Triangle

Each point *A*, *B*, and *C* is a **vertex** of the triangle; collectively, these three points are the **vertices** of the triangle.

$\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are the **sides** of the triangle.

Point *D* is in the **interior** of the triangle; point *E* is on the triangle; and point *F* is in the **exterior** of the triangle.

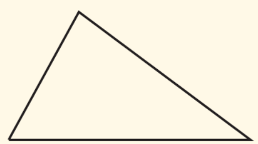
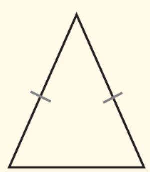
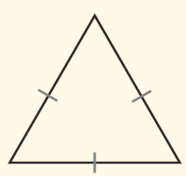
Triangles may be categorized by the lengths of their sides.

# The Angles of a Triangle

Table 2.1 presents each type of triangle, the relationship among its sides, and a drawing in which congruent sides are marked.

**TABLE 2.1**

***Triangles Classified by Congruent Sides***

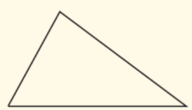
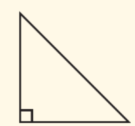
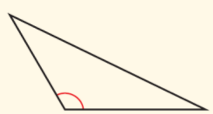
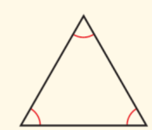
| Type | | Number of Congruent Sides |
|---|---|---|
| Scalene |  | None |
| Isosceles |  | Two |
| Equilateral |  | Three |

# The Angles of a Triangle

Triangles may also be classified according to the measures of their angles as shown in Table 2.2.

**TABLE 2.2**

***Triangles Classified by Angles***

| Type | | Angle(s) | Type | | Angle(s) |
|---|---|---|---|---|---|
| Acute |  | All angles acute | Right |  | One right angle |
| Obtuse |  | One obtuse angle | Equiangular |  | All angles congruent |

# Example 1

In $\Delta HJK$ (not shown), $HJ = 4$, $JK = 4$, and $m\angle J = 90°$. Describe completely the type of triangle represented.

Solution:

$\Delta HJK$ is a right isosceles triangle, or $\Delta HJK$ is an isosceles right triangle.

# The Angles of a Triangle

The sum of the measures of the three interior angles of a triangle is 180°. This is proved through the use of an **auxiliary** (or helping) **line.**

When an auxiliary line is added to the drawing for a proof, a *justification* must be given for the existence of that line.

Justifications include statements such as

*There is exactly one line through two distinct points.*

*An angle has exactly one bisector.*

*There is only one line perpendicular to another line at a point on that line.*

# The Angles of a Triangle

When an auxiliary line is introduced into a proof, the original drawing is redrawn for the sake of clarity.

Each auxiliary figure must be **determined,** but not **underdetermined** or **overdetermined.**

A figure is underdetermined when more than one figure is possible.

On the other extreme, a figure is overdetermined when it is impossible for the drawing to include *all* conditions described.

# The Angles of a Triangle

Theorem 2.4.1

In a triangle, the sum of the measures of the interior angles is 180°.

A theorem that follows directly from a previous theorem is known as a **corollary** of that theorem.

Corollary 2.4.2

Each angle of an equiangular triangle measures 60°.

# The Angles of a Triangle

Corollary 2.4.3

The acute angles of a right triangle are complementary.

Corollary 2.4.4

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

# The Angles of a Triangle

When the sides of a triangle are extended, each angle that is formed by a side and an extension of the adjacent side is an **exterior angle** of the triangle.

With *B*-*C*-*D* in Figure 2.26(a), $\angle ACD$ is an exterior angle of $\Delta ABC$; for a triangle, there are a total of six exterior angles—two at each vertex. [See Figure 2.26(b).]

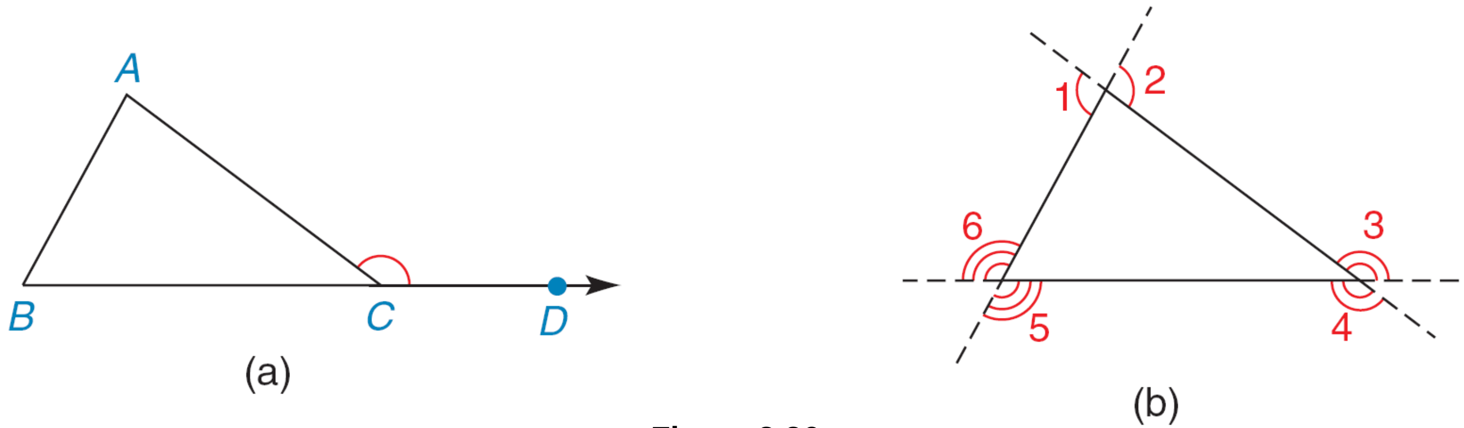



**Figure 2.26**

# The Angles of a Triangle

In Figure 2.26(a), $\angle A$ and $\angle B$ are the two *nonadjacent* interior angles for exterior $\angle ACD$.

These angles (*A* and *B*) are sometimes called *remote* interior angles for exterior $\angle ACD$.

Corollary 2.4.5

The measure of an exterior angle of a triangle equals the sum of the measures of the two nonadjacent interior angles.