Chapter 2 Parallel Lines

2.3 Proving Lines Parallel

Proving Lines Parallel

Postulate 11

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Theorem 2.1.2

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Theorem 2.1.3

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

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Theorem 2.1.4

If two parallel lines are cut by a transversal, then the pairs of interior angles on the same side of the transversal are supplementary.

Theorem 2.1.5

If two parallel lines are cut by a transversal, then the pairs of exterior angles on the same side of the transversal are supplementary.

Proving Lines Parallel

Theorem 2.3.1

If two lines are cut by a transversal so that two corresponding angles are congruent, then these lines are parallel.

Given:

ℓ and m cut by transversal t

$\angle 1 \cong \angle 2$ (See Figure 2.16)

Prove:

$\ell \parallel m$

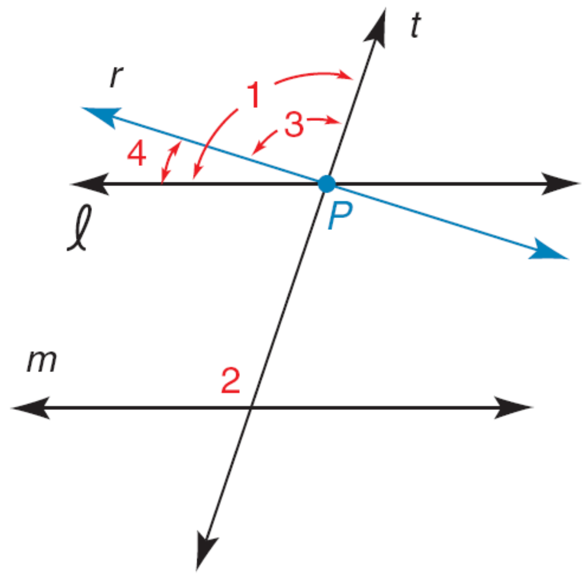

Figure 2.16

Proving Lines Parallel

Theorem 2.3.2

If two lines are cut by a transversal so that two alternate interior angles are congruent, then these lines are parallel.

Given:

Lines ℓ and m and transversal t

$\angle 2 \cong \angle 3$ (See Figure 2.17)

Prove:

$\ell \parallel m$

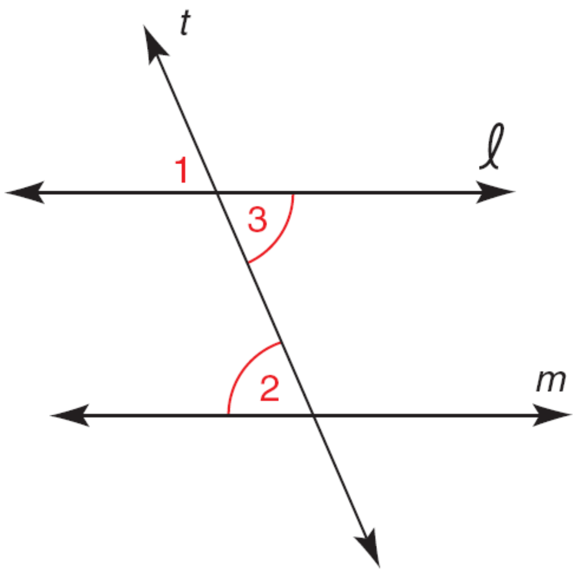

Figure 2.17

Proving Lines Parallel

Theorem 2.3.3

If two lines are cut by a transversal so that two alternate exterior angles are congruent, then these lines are parallel.

In a more complex drawing, it may be difficult to decide which lines are parallel because of congruent angles.

Consider Figure 2.18.

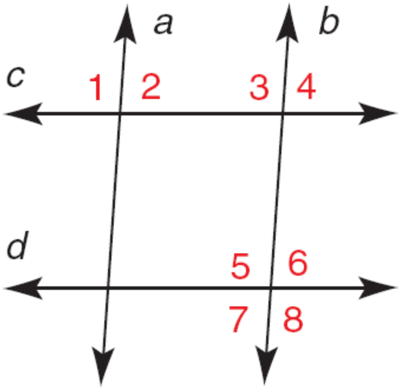

Figure 2.18

Proving Lines Parallel

Suppose that $\angle 1 \cong \angle 3$.

Which lines must be parallel?

The resulting confusion (it appears that *a* may be parallel to *b* and *c* may be parallel to *d*) can be overcome by asking, "Which lines help form $\angle 1$ and $\angle 3$?" In this case, $\angle 1$ and $\angle 3$ are formed by lines *a* and *b* with *c* as the transversal.

Thus, $a \parallel b$.

Example 1

In Figure 2.18, which lines must be parallel if $\angle 3 \cong \angle 8$?

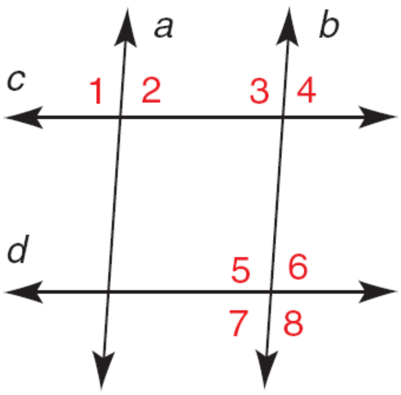

Figure 2.18

Solution:

$\angle 3$ and $\angle 8$ are the alternate exterior angles formed when lines *c* and *d* are cut by transversal *b.*

Thus, $c \parallel d$.

Proving Lines Parallel

Theorem 2.3.4

If two lines are cut by a transversal so that two interior angles on the same side of the transversal are supplementary, then these lines are parallel.

Theorem 2.3.5

If two lines are cut by a transversal so that two exterior angles on the same side of the transversal are supplementary, then the lines are parallel.

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Theorem 2.3.6

If two lines are each parallel to a third line, then these lines are parallel to each other.

Theorem 2.3.7

If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

Proving Lines Parallel

Construction 7

To construct the line parallel to a given line from a point not on that line.

Given: $\overleftrightarrow{AB}$ and point P not on $\overleftrightarrow{AB}$, as in Figure 2.23(a)

Figure 2.23 (a)

Proving Lines Parallel

Construct: The line through point *P* parallel to $\overleftrightarrow{AB}$

Construction: Figure 2.23(b): Draw a line (to become a transversal) through point *P* and some point on $\overleftrightarrow{AB}$.

For convenience, we choose point *A* and draw $\overleftrightarrow{AP}$.

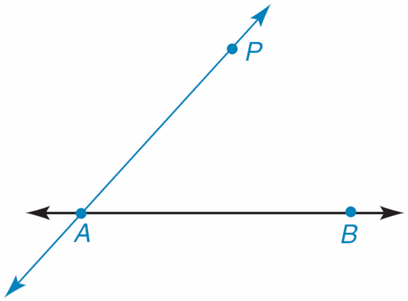

Figure 2.23(b)

Proving Lines Parallel

Figure 2.23(c): Using *P* as the vertex, construct the angle that corresponds to $\angle PAB$ so that this angle is congruent to $\angle PAB$.

It may be necessary to extend $\overleftrightarrow{AP}$ upward to accomplish this. $\overleftrightarrow{PX}$ is the desired line parallel to $\overleftrightarrow{AB}$.

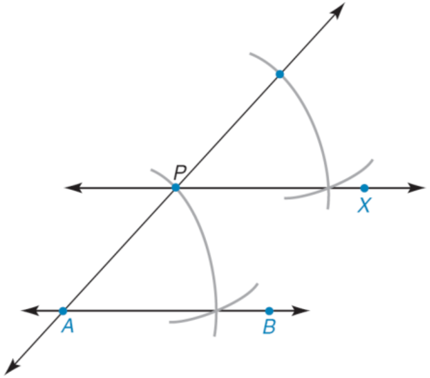

Figure 2.23(c)