Chapter 2 Parallel Lines

2.2 Indirect Proof

Indirect Proof

Let $P \rightarrow Q$ represent the *conditional* statement "If P, then Q." The following statements are related to this conditional statement (also called an *implication*).

Note: Recall that $\sim P$ represents the negation of P.

Conditional (or Implication)	$P \rightarrow Q$	If P, then Q.
Converse of Conditional	$Q \rightarrow P$	If Q, then P.
Inverse of Conditional	$\sim P \rightarrow \sim Q$	If not P, then not Q.
Contrapositive of Conditional	$\sim Q \rightarrow \sim P$	If not Q, then not P.

Indirect Proof

Consider the following conditional statement.

If Tom lives in San Diego, then he lives in California.

This true conditional statement has these related statements:

Converse: If Tom lives in California, then he lives in San Diego. (false)

Inverse: If Tom does not live in San Diego, then he does not live in California. (false)

Contrapositive: If Tom does not live in California, then he does not live in SanDiego. (true)

Indirect Proof

In general, the conditional statement and its contrapositive are either both true or both false! Similarly, the converse and the inverse are also either both true or both false.

Example 1

For the conditional statement that follows, give the converse, the inverse, and the contrapositive. Then classify each as true or false.

If two angles are vertical angles, then they are congruent angles.

Example 1 – *Solution*

Converse:	If two angles are congruent angles, then they are vertical angles. (false)
Inverse:	If two angles are not vertical angles, then they are not congruent angles. (false)
Contrapositive:	If two angles are not congruent angles, then they are not vertical angles. (true)

Indirect Proof

> "If P, then Q" and "If not Q, then not P" are equivalent.

Venn Diagrams can be used to explain why the conditional statement $P \rightarrow Q$ and its contrapositive $\sim Q \rightarrow \sim P$ are equivalent. The relationship "If *P*, then *Q*" is represented in Figure 2.11.

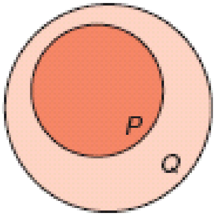

Figure 2.11

Note that if any point is selected outside of *Q* (that is, $\sim Q$), then it cannot possibly lie in set P (thus, $\sim P$).

THE LAW OF NEGATIVE INFERENCE (CONTRAPOSITION)

The Law of Negative Inference (Contraposition)

Consider the following circumstances, and accept each premise as true:

1. If Matt cleans his room, then he will go to the movie. ($P \rightarrow Q$)

2. Matt does not get to go to the movie. ($\sim Q$)

The Law of Negative Inference (Contraposition)

What can you conclude? You should have deduced that Matt did not clean his room; if he had, he would have gone to the movie.

This "backdoor" reasoning is based on the fact that the truth of $P \to Q$ implies the truth of $\sim Q \to \sim P$.

Law of Negative Inference (Contraposition)

1.	$P \to Q$
2.	$\sim Q$
C.	$\therefore \sim P$

INDIRECT PROOF

Indirect Proof

You will need to know when to use the indirect method of proof. Often the theorem to be proved has the form $P \rightarrow Q$, in which Q is a negation and denies some claim.

For instance, an indirect proof might be best if Q reads in one of these ways:

c is *not* equal to d

ℓ is *not* perpendicular to m

However, we will see in next example that the indirect method can be used to prove that line ℓ is parallel to line m.

Example 5

Given: In Figure 2.14, plane T intersects parallel planes P and Q in lines ℓ and m, respectively

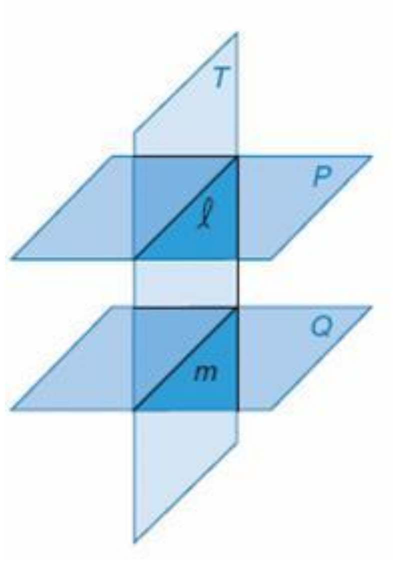

Figure 2.14

Prove: $\ell \parallel m$

Example 5

cont'd

Proof:

Assume that ℓ is not parallel to m.

Then ℓ and m intersect at some point A.

But if so, point A must be on both planes P and Q, which means that planes P and Q intersect; but P and Q are parallel by hypothesis.

Therefore, the assumption that ℓ and m are not parallel must be false, and it follows that $\ell \parallel m$.