Chapter 2 Parallel Lines

2.1 The Parallel Postulate and Special Angles

PERPENDICULAR LINES

Perpendicular Lines

By definition, two lines (or segments or rays) are perpendicular if they meet to form congruent adjacent angles.

Using this definition, we proved the theorem stating that "perpendicular lines meet to form right angles."

We can also say that two rays or line segments are perpendicular if they are parts of perpendicular lines.

We now consider a method for constructing a line perpendicular to a given line.

Perpendicular Lines

Construction 6

To construct the line that is perpendicular to a given line from a point not on the given line.

Given: In Figure 2.1(a), line ℓ and point *P* not on ℓ

Construct: $\overleftrightarrow{PQ} \perp \ell$

Construction: Figure 2.1(b): With *P* as the center, open the compass to a length great enough to intersect ℓ in two points *A* and *B.*

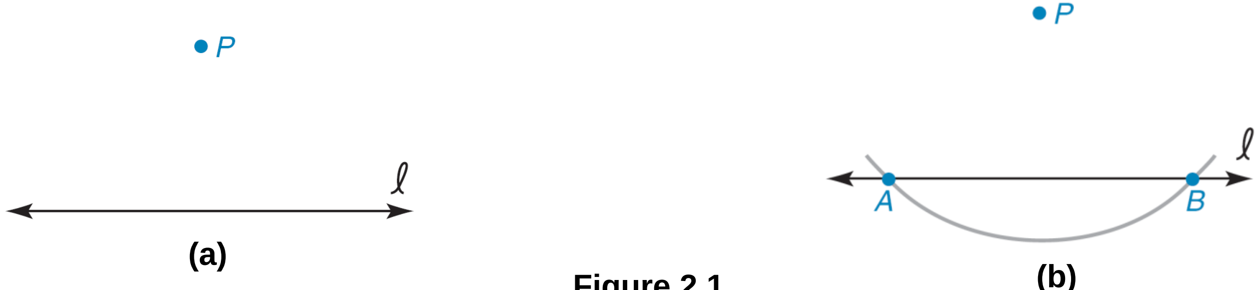


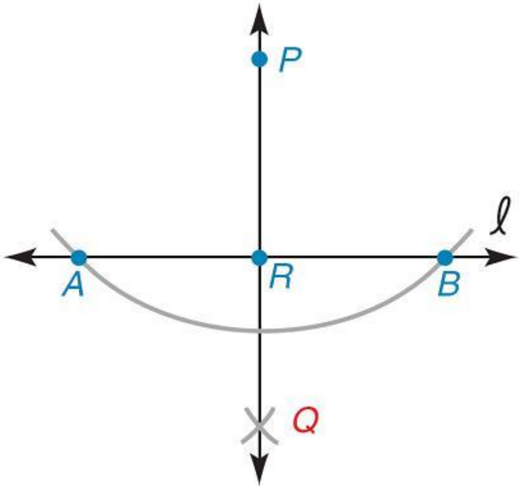
Figure 2.1

Perpendicular Lines

Figure 2.1(c): With *A* and *B* as centers, mark off arcs of equal radii (using the same compass opening) to intersect at a point *Q*, as shown.

Draw $\overleftrightarrow{PQ}$ to complete the desired line.

In this construction, $\angle PRA$ and $\angle PRB$ are right angles. Greater accuracy is achieved if the arcs drawn from *A* and *B* intersect on the opposite side of line ℓ from point *P*.



(c)

Figure 2.1

Perpendicular Lines

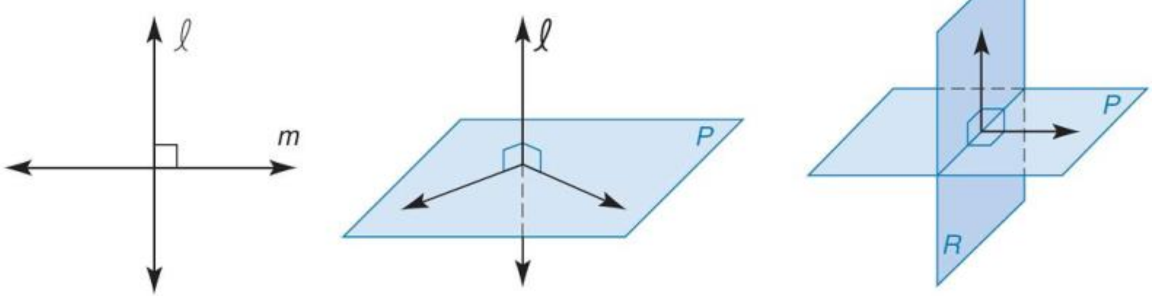
Theorem 2.1.1

From a point not on a given line, there is exactly one line perpendicular to the given line.

The term *perpendicular* includes line-ray, line-plane, and plane-plane relationships.

Perpendicular Lines

In Figure 2.1(c), $\overrightarrow{RP} \perp \ell$. The drawings in Figure 2.2 indicate two perpendicular lines, a line perpendicular to a plane, and two perpendicular planes.



(a) $\ell \perp m$ (b) $\ell \perp P$ (c) $R \perp P$

Figure 2.2

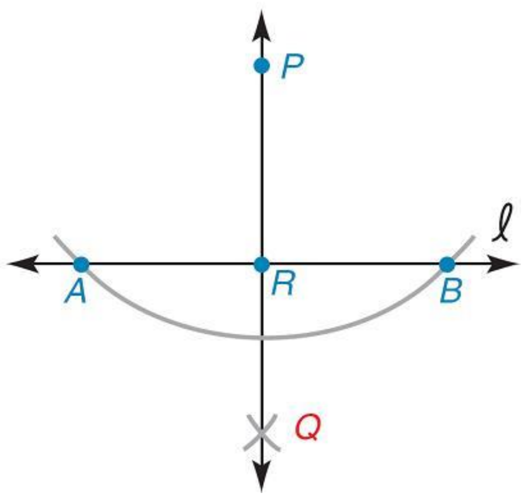


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Figure 2.1

PARALLEL LINES

Parallel Lines

Just as the word *perpendicular* can relate lines and planes, the word *parallel* can also be used to describe relationships among lines and planes. However, parallel lines must lie in the same plane, as the following definition emphasizes.

Definition

Parallel lines are lines in the same plane that do not intersect.

Parallel Lines

More generally, two lines in a plane, a line and a plane, or two planes are parallel if they do not intersect. Figure 2.3 illustrates possible applications of the word *parallel*.

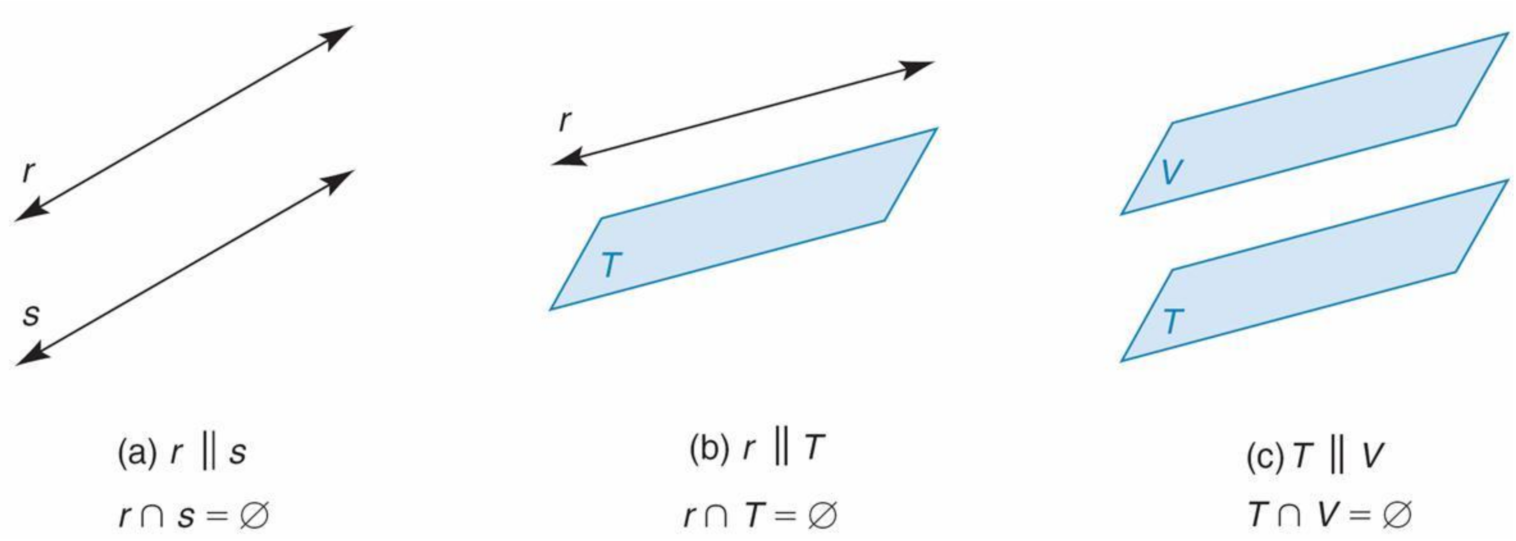


Figure 2.3

Parallel Lines

In Figure 2.4, two parallel planes *M* and *N* are both intersected by a third plane *G*.

How must the lines of intersection, *a* and *b*, be related?

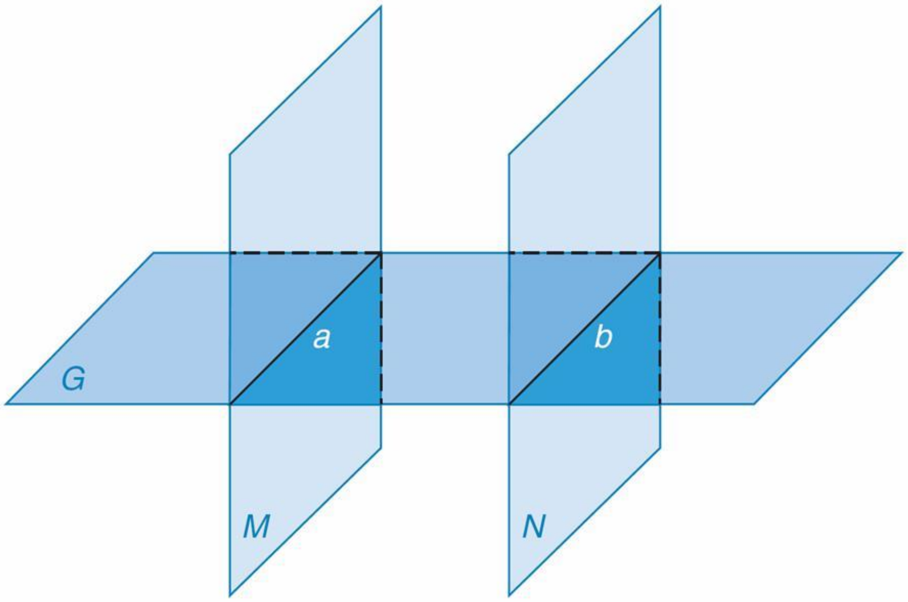


Figure 2.4

EUCLIDEAN GEOMETRY

Euclidean Geometry

In this geometry, we interpret a plane as a flat, two-dimensional surface in which the line segment joining any two points of the plane lies entirely within the plane.

Whereas the postulate that follows characterizes Euclidean geometry, the Perspective on Applications section near the end of this chapter discusses alternative geometries.

Postulate 10, the Euclidean Parallel Postulate, is easy to accept because of the way we perceive a plane.

Euclidean Geometry

Postulate 10 (Parallel Postulate)

Through a point not on a line, exactly one line is parallel to the given line.

Consider Figure 2.5, in which line *m* and point *P* (with *P* not on *m*) both lie in plane *R*. It seems reasonable that exactly one line can be drawn through *P* parallel to line *m*.

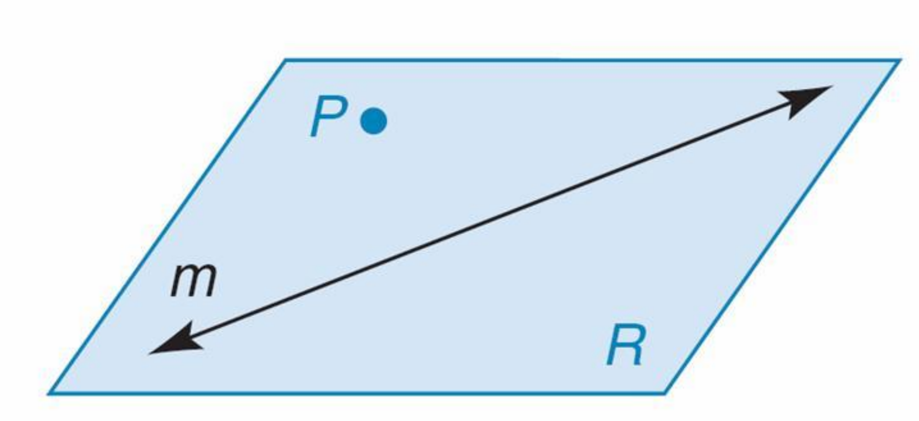


Figure 2.5

Euclidean Geometry

A **transversal** is a line that intersects two (or more) other lines at distinct points; all of the lines lie in the same plane. In Figure 2.6, *t* is a transversal for lines *r* and *s*.

Angles that are formed between *r* and *s* are **interior angles**; those outside *r* and *s* are **exterior angles**.

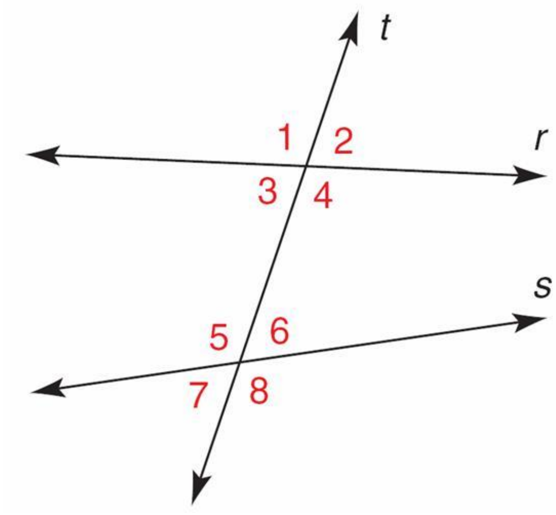


Figure 2.6

Euclidean Geometry

Relative to Figure 2.6, we have

Interior angles: $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior angles: $\angle 1, \angle 2, \angle 7, \angle 8$

Consider the angles in Figure 2.6 that are formed when lines are cut by a transversal. Two angles that lie in the same relative positions (such as *above* and *left*) are called **corresponding angles** for these lines.

In Figure 2.6, $\angle 1$ and $\angle 5$ are corresponding angles; each angle is *above* the line and to the *left* of the transversal that together form the angle.

Euclidean Geometry

As shown in Figure 2.6, we have

Corresponding angles:

(must be in pairs)

$\angle 1$ and $\angle 5$ above left

$\angle 3$ and $\angle 7$ below left

$\angle 2$ and $\angle 6$ above right

$\angle 4$ and $\angle 8$ below right

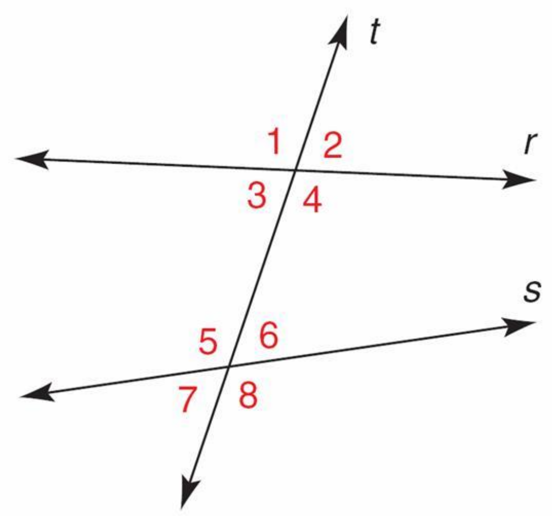


Figure 2.6

Two interior angles that have different vertices and lie on opposite sides of the transversal are **alternate interior angles**.

Euclidean Geometry

Two exterior angles that have different vertices and lie on opposite sides of the transversal are **alternate exterior angles**.

Both types of alternate angles must occur in pairs; in Figure 2.6, we have:

Alternate interior angles: $\angle 3$ and $\angle 6$
(must be in pairs) $\angle 4$ and $\angle 5$

Alternate exterior angles: $\angle 1$ and $\angle 8$
(must be in pairs) $\angle 2$ and $\angle 7$

PARALLEL LINES AND CONGRUENT ANGLES

Parallel lines and Congruent angles

In Figure 2.7, *parallel* lines ℓ and m are cut by transversal v. If a protractor were used to measure $\angle 1$ and $\angle 5$, these corresponding angles would have equal measures; that is, they would be congruent.

Similarly, any other pair of corresponding angles would be congruent as long as $\ell \parallel m$.

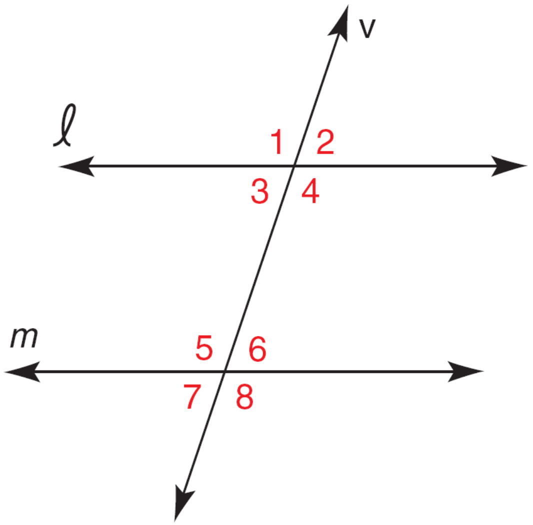


Figure 2.7

Parallel lines and Congruent angles

Postulate 11

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Example 1

In Figure 2.7, $\ell \parallel m$ and m $\angle 1 = 117°$. Find:

a) m$\angle 2$ **c)** m$\angle 4$

b) m$\angle 5$ **d)** m$\angle 8$

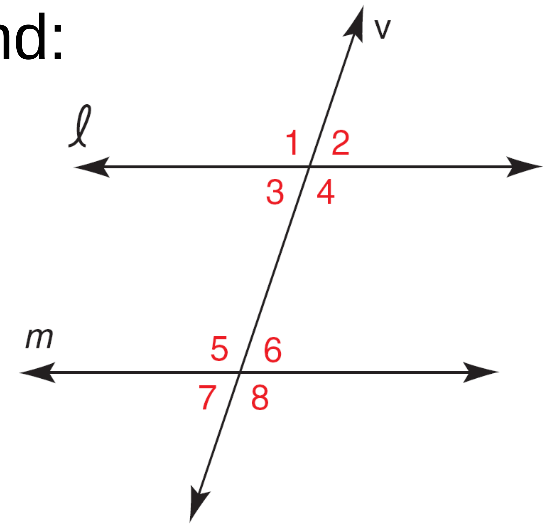


Figure 2.7

Solution:

a) m$\angle 2 = 63°$ supplementary to $\angle 1$

b) m$\angle 5 = 117°$ corresponding to $\angle 1$

c) m$\angle 4 = 117°$ vertical to $\angle 1$

d) m$\angle 8 = 117°$ corresponding to $\angle 4$ [found in part (c)]

Parallel lines and Congruent angles

Theorem 2.1.2

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Given: $a \parallel b$ in Figure 2.8

Transversal k

Prove: $\angle 3 \cong \angle 6$

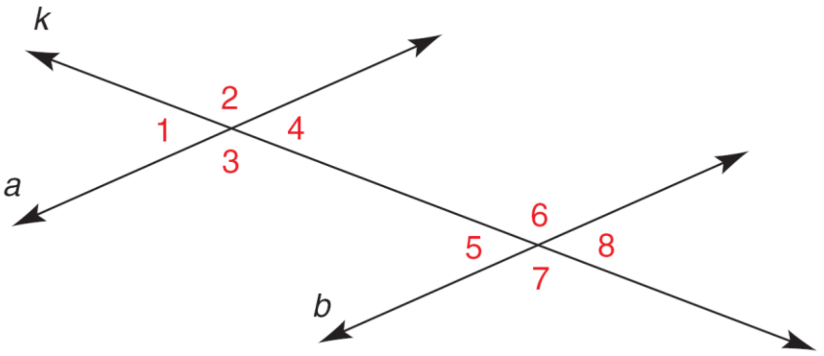


Figure 2.8

Parallel lines and Congruent angles

Theorem 2.1.3

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

PARALLEL LINES AND SUPPLEMENTARY ANGLES

Parallel lines and Supplementary angles

When two parallel lines are cut by a transversal, it can be shown that the two interior angles on the same side of the transversal are supplementary.

A similar claim can be made for the pair of exterior angles on the same side of the transversal.

Parallel lines and Supplementary angles

Theorem 2.1.4

If two parallel lines are cut by a transversal, then the pairs of interior angles on the same side of the transversal are supplementary.

Given: In Figure 2.9, $\overleftrightarrow{TV} \parallel \overleftrightarrow{WY}$ with transversal $\overleftrightarrow{RS}$

Prove: $\angle 1$ and $\angle 3$ are supplementary

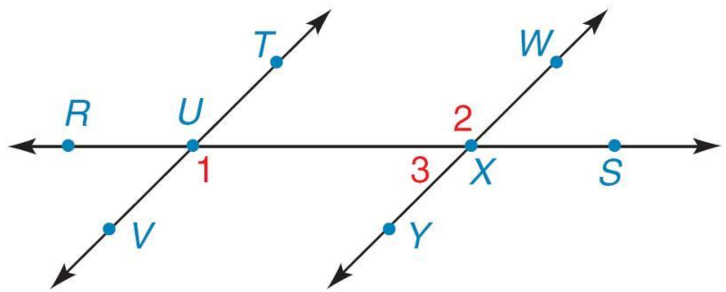


Figure 2.9

Parallel lines and Supplementary angles

Theorem 2.1.5

If two parallel lines are cut by a transversal, then the pairs of exterior angles on the same side of the transversal are supplementary.

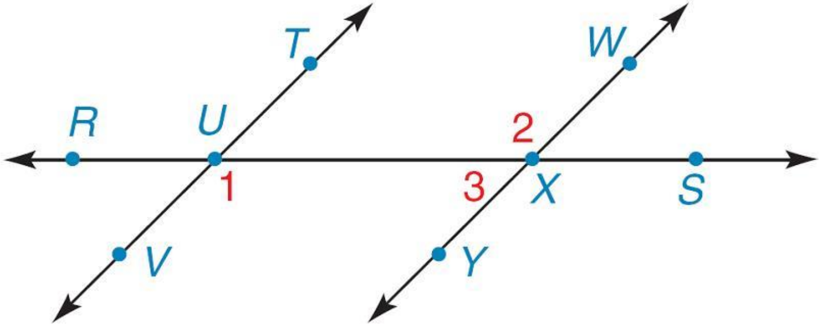
Example 2

Given: $\overleftrightarrow{TV} \parallel \overleftrightarrow{WY}$ with transversal $\overleftrightarrow{RS}$

$$m\angle RUV = (x + 4)(x - 3)$$

$$m\angle WXS = x^2 - 3$$

Find: x



Example 2 – *Solution*

$\angle RUV$ and $\angle WXS$ are alternate exterior angles, so they are congruent.

Then $m\angle RUV = m\angle WXS$. Therefore,

$$(x + 4)(x - 3) = x^2 - 3$$

$$x^2 + x - 12 = x^2 - 3$$

$$x - 12 = -3$$

$$x = 9$$

Note: $\angle RUV$ and $\angle WXS$ both measure 78° when $x = 9$.