Chapter 11 Introduction to Trigonometry

11.3 The Tangent Ratio and Other Ratios

The Tangent Ratio and Other Ratios

The next trigonometric ratio that we consider is the **tangent** ratio, which is defined for an acute angle of the right triangle by

$$\frac{\text{length of leg } \textit{opposite} \text{ acute angle}}{\text{length of leg } \textit{adjacent} \text{ to acute angle}}$$

Like the sine ratio, the tangent ratio increases as the measure of the acute angle increases.

Unlike the sine and cosine ratios, whose values range from 0 to 1, the value of the tangent ratio is from 0 upward; that is, there is no greatest value for the tangent.

The Tangent Ratio and Other Ratios

Definition

In a right triangle, the **tangent ratio** for an acute angle is the ratio $\frac{\text{opposite}}{\text{adjacent}}$.

Example 1

Find the values of tan α and tan β for the triangle in Figure 11.25.

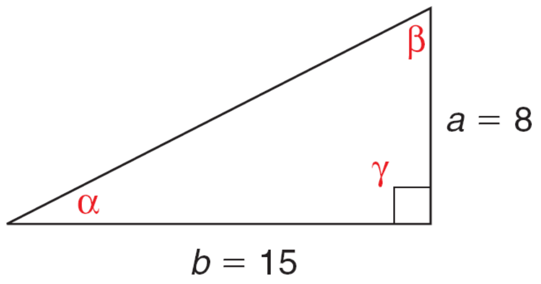

Figure 11.25

Solution:

Using the fact that the tangent ratio is $\frac{\text{opposite}}{\text{adjacent}}$, we find that

$$\tan \alpha = \frac{a}{b} = \frac{8}{15}$$

and

$$\tan \beta = \frac{b}{a} = \frac{15}{8}$$

The Tangent Ratio and Other Ratios

The value of tan θ changes from 0 for a 0° angle to an immeasurably large value as the measure of the acute angle approaches 90°.

That the tangent ratio $\frac{\text{opposite}}{\text{adjacent}}$ becomes infinitely large as $\theta \to 90°$ follows from the fact that the denominator becomes smaller (approaching 0) while the numerator increases.

The Tangent Ratio and Other Ratios

Study Figure 11.26 to see why the value of the tangent of an angle grows immeasurably large as the angle approaches 90° in size.

We often express this relationship by writing: As $\theta \to 90°$, tan $\theta \to \infty$.

The symbol ∞ is read "infinity" and implies that tan 90° is not measurable; thus, tan 90° is *undefined*.

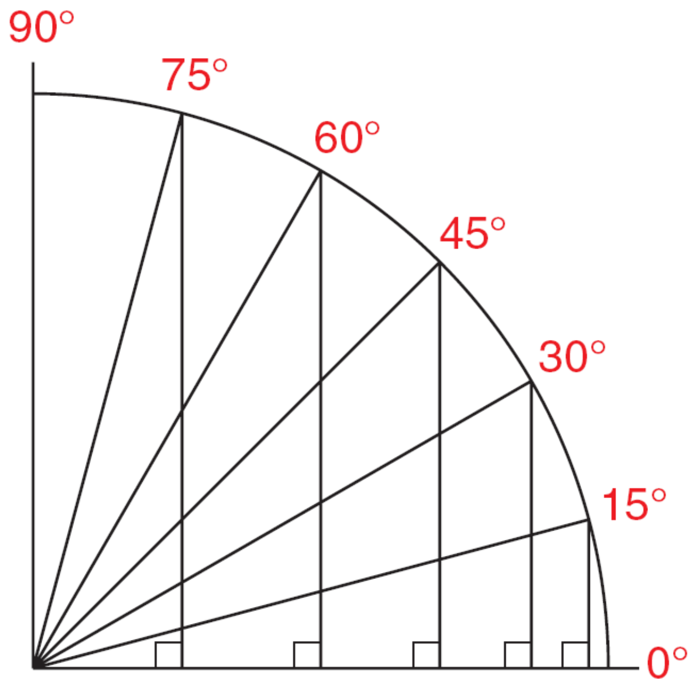

Figure 11.26

The Tangent Ratio and Other Ratios

Definition

tan 0° = 0 and tan 90° is undefined.

Certain tangent ratios are found by using special right triangles. By observing the triangles in Figure 11.27 and using the fact that, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$, we have

$$\tan 30^\circ = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.5774$$

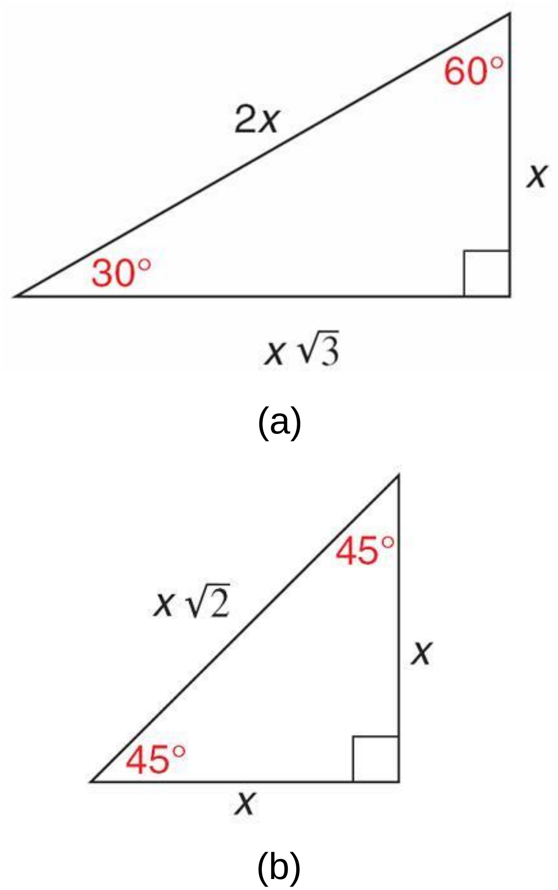

Figure 11.27

The Tangent Ratio and Other Ratios

$$\tan 45° = \frac{x}{x} = 1$$

$$\tan 60° = \frac{x\sqrt{3}}{x} = \sqrt{3} \approx 1.7321$$

The tangent ratio can also be used to find the measure of an angle if the lengths of the legs of a right triangle are known.

The Tangent Ratio and Other Ratios

For the right triangle in Figure 11.30, we now have three ratios that can be used in problem solving.

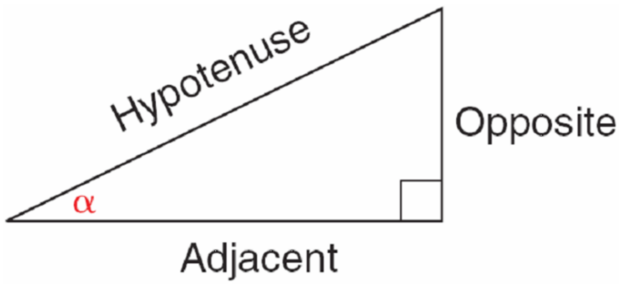

Figure 11.30

These are summarized as follows:

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

The Tangent Ratio and Other Ratios

The equation $\tan \alpha = \frac{a}{b}$ contains three variables: α, *a*, and *b*.

If the values of two of the variables are known, the value of the third variable can be found.

There are a total of six trigonometric ratios.

We define the remaining ratios for completeness; however, we will be able to solve all application problems by using only the sine, cosine, and tangent ratios.

The Tangent Ratio and Other Ratios

The remaining ratios are the **cotangent** (abbreviated “cot”), **secant** (abbreviated “sec”), and **cosecant** (abbreviated “csc”).

These are defined in terms of the right triangle shown in Figure 11.33.

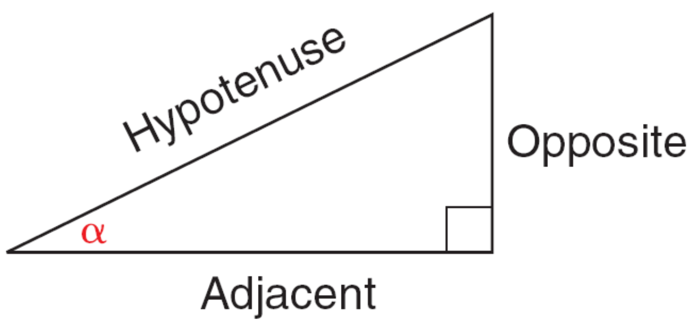

Figure 11.33

The Tangent Ratio and Other Ratios

$$\cot \alpha = \frac{\text{adjacent}}{\text{opposite}}$$
$$\sec \alpha = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\csc \alpha = \frac{\text{hypotenuse}}{\text{opposite}}$$

For the fraction $\frac{a}{b}$ (where $b \neq 0$), the reciprocal is $\frac{b}{a}$ ($a \neq 0$). It is easy to see that cot α is the reciprocal of tan α, sec α is the reciprocal of cos α, and csc α is the reciprocal of sin α.

The Tangent Ratio and Other Ratios

In Table 11.4, we invert the trigonometric ratio on the left to obtain the reciprocal ratio named to its right.

TABLE 11.4

The Six Trigonometric Ratios

$$\text{sine } \alpha = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosecant } \alpha = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{cosine } \alpha = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{secant } \alpha = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{tangent } \alpha = \frac{\text{opposite}}{\text{adjacent}} \qquad \text{cotangent } \alpha = \frac{\text{adjacent}}{\text{opposite}}$$