Chapter 10 Analytic Geometry

10.5 Equations of Lines

Equations of Lines

We know that equations such as $2x + 3y = 6$ and $4x - 12y = 60$ have graphs that are lines.

To graph an equation of the general form $Ax + By = C$, that equation is often replaced with an equivalent equation of the form $y = mx + b$.

Equations of Lines

For instance, $2x + 3y = 6$ can be transformed into $y = -\frac{2}{3}x + 2$; equations such as these are known as *equivalent* because their ordered-pair solutions (and graphs) are identical.

In particular, we must express a linear equation in the form $y = mx + b$ in order to plot it on a graphing calculator.

Example 1

Write the equation $4x - 12y = 60$ in the form $y = mx + b$.

Solution:

Given $4x - 12y = 60$, we subtract $4x$ from each side of the equation to obtain $-12y = -4x + 60$.

Dividing by –12,

$$\frac{-12y}{-12} = \frac{-4x}{-12} + \frac{60}{-12}$$

Then $y = \frac{1}{3}x - 5$.

SLOPE-INTERCEPT FORM OF A LINE

Slope-Intercept Form of a Line

We now turn our attention to a method for finding the equation of a line. In the following technique, the equation can be found if the slope m and the y intercept b of the line are known.

The form $y = mx + b$ is known as the Slope-Intercept Form of a line.

Theorem 10.5.1 (Slope-intercept Form of a Line)

The line whose slope is m and whose y intercept is b has the equation $y = mx + b$.

POINT-SLOPE FORM OF A LINE

Point-Slope Form of a Line

If slope m and a point other than the y intercept of a line are known, we generally do not use the Slope-Intercept Form to find the equation of the line. Instead, the Point-Slope Form of the equation of a line is used.

This form is also used when the coordinates of two points of the line are known; in that case, the value of m is found by the Slope Formula.

The form $y - y_1 = m(x - x_1)$ is known as the Point-Slope Form of a line.

Point-Slope Form of a Line

Theorem 10.5.2 (Point-Slope Form of a Line)

The line that has slope m and contains the point (x_1, y_1) has the equation

$$y - y_1 = m(x - x_1)$$

SOLVING SYSTEMS OF EQUATIONS

Example 7

Solve the following system by using algebra:

$$\begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases}$$

Solution:

When we multiply the second equation by 2, the system becomes

$$\begin{cases} x + 2y = 6 \\ 4x - 2y = 14 \end{cases}$$

Example 7 – *Solution*

cont'd

Adding these equations yields $5x = 20$, so $x = 4$.

Substituting $x = 4$ into the first equation, we find that $4 + 2y = 6$, so $2y = 2$.

Then $y = 1$.

The solution is the ordered pair (4, 1).

Solving Systems of Equations

Advantages of the method of solving a system of equations by graphing include the following:

1. It is easy to understand why a system such as

$$\begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases} \quad \text{can be replaced by} \quad \begin{cases} x + 2y = 6 \\ 4x - 2y = 14 \end{cases}$$

when we are solving by addition or subtraction. We know that the graphs of $2x - y = 7$ and $4x - 2y = 14$ are coincident (the same line) because each can be changed to the form $y = 2x - 7$.

Solving Systems of Equations

2. It is easy to understand why a system such as

$$\begin{cases} x + 2y = 6 \\ 2x + 4y = -4 \end{cases}$$

has no solution.

In Figure 10.40, the graphs of these equations are parallel lines.

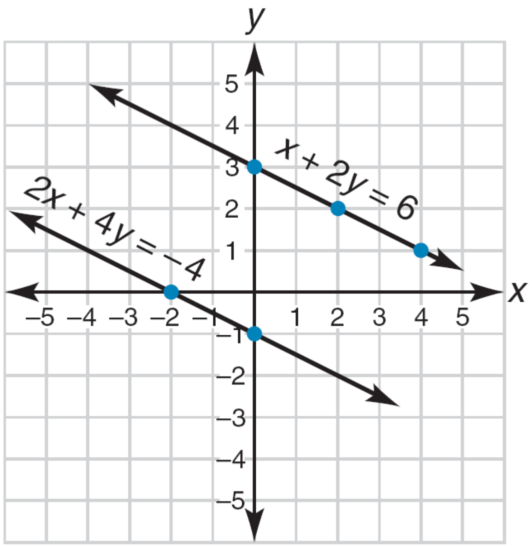

Figure 10.40

Solving Systems of Equations

The first equation is equivalent to $y = -\frac{1}{2}x + 3$, and the second equation can be changed to $y = -\frac{1}{2}x - 1$. Both lines have slope $m = -\frac{1}{2}$ but have different *y* intercepts. Therefore, the lines are parallel and distinct.

Algebraic substitution can also be used to solve a system of equations. In our approach, we write each equation in the form $y = mx + b$ and then equate the expressions for *y*.

Once the *x* coordinate of the solution is known, we substitute this value of *x* into either equation to find the value of *y*.

Solving Systems of Equations

Theorem 10.5.3

The three medians of a triangle are concurrent at a point that is two-thirds the distance from any vertex to the midpoint of the opposite side.