Chapter 11 Introduction to Trigonometry

11.2 The Cosine Ratio and Applications

The Cosine Ratio and Applications

Again we deal strictly with similar right triangles. In Figure 11.13, $\angle A \cong \angle D$ and $\angle C \cong \angle F$; thus, $\triangle ABC \sim \triangle DEF$ by AA.

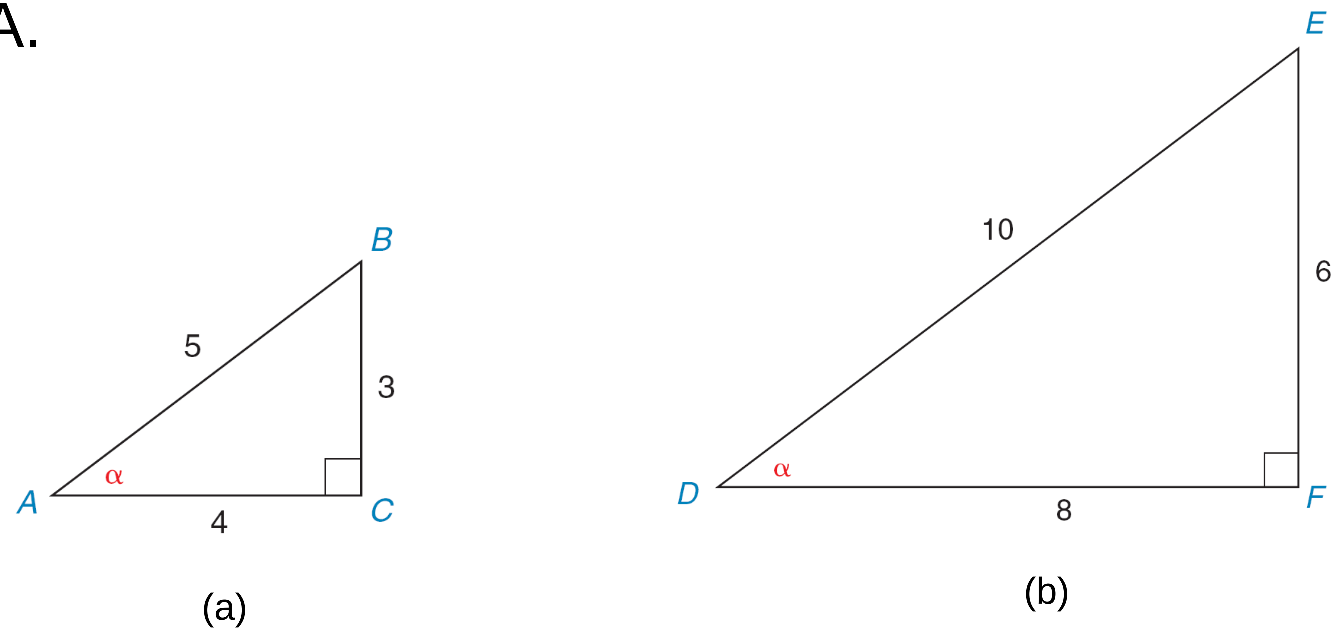

Figure 11.13

In $\triangle ABC$, $\overline{BC}$ is the leg opposite angle *A*, while $\overline{AC}$ is the leg *adjacent* to angle *A*.

The Cosine Ratio and Applications

In the two triangles, the ratios of the form

$$\frac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$$

are equal; that is,

$$\frac{AC}{AB} = \frac{DF}{DE} \quad \text{or} \quad \frac{4}{5} = \frac{8}{10}$$

This relationship follows from the fact that corresponding sides of similar triangles are proportional (CSSTP).

As with the sine ratio, the *cosine ratio* depends on the measure of acute angle *A* (or *D*) in Figure 11.13.

The Cosine Ratio and Applications

In the following definition, the term *adjacent* refers to the length of the leg that is adjacent to the angle named.

Definition

In a right triangle, the **cosine ratio** for an acute angle is the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Example 1

Find cos α and cos β for right $\triangle ABC$ in Figure 11.15.

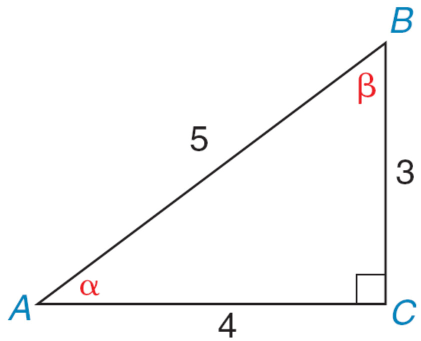

Figure 11.15

Solution:

a = 3, b = 4, and c = 5 for the triangle shown in Figure 11.15.

Because b is the length of the leg adjacent to α and a is the length of the leg adjacent to β,

$$\cos\alpha = \frac{b}{c} = \frac{4}{5} \quad \text{and} \quad \cos\beta = \frac{a}{c} = \frac{3}{5}$$

The Cosine Ratio and Applications

Just as the sine ratio of any angle is unique, the cosine ratio of any angle is also unique.

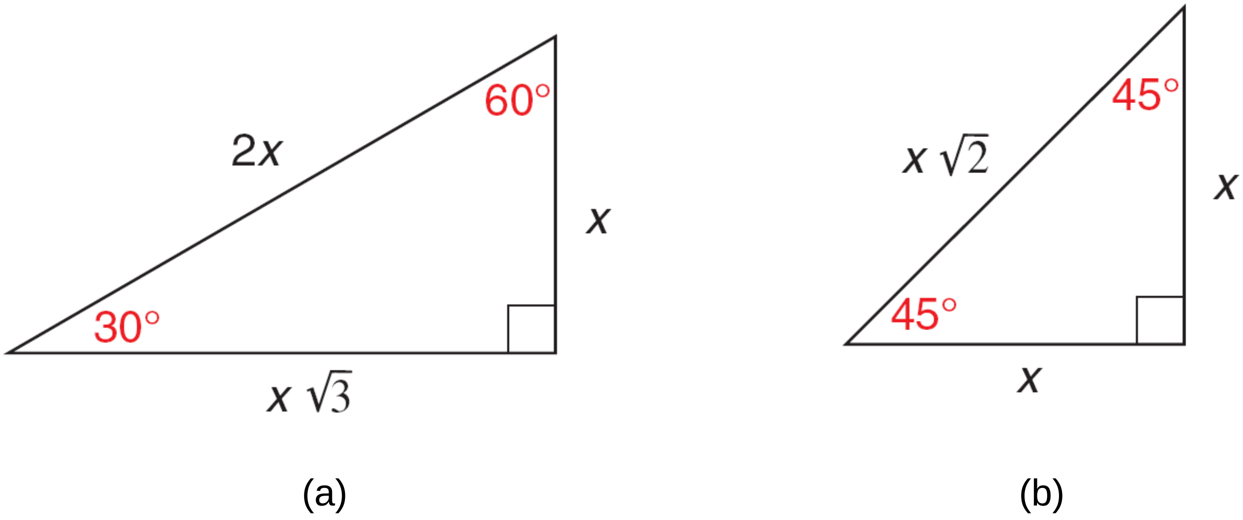

Figure 11.17

The Cosine Ratio and Applications

Using the 30°-60°-90° and 45°-45°-90° triangles of Figure 11.17, we see that

$$\cos 30° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 45° = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

$$\cos 60° = \frac{x}{2x} = \frac{1}{2} = 0.5$$

The Cosine Ratio and Applications

Now we use the 15°-75°-90° triangle shown in Figure 11.18 to find cos 75° and cos 15°.

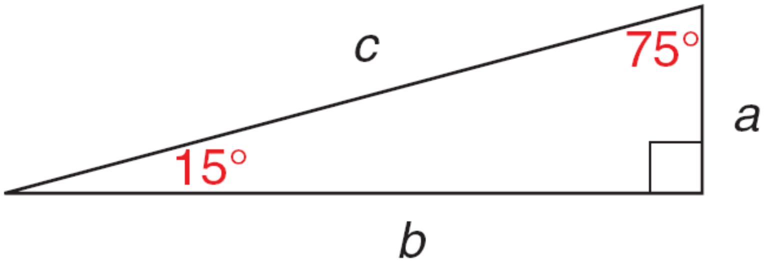

Figure 11.18

sin 15° = $\frac{a}{c}$ and sin 15° = 0.2588.

But cos 75° = $\frac{a}{c}$, so cos 75° = 0.2588.

Similarly, because sin 75° = $\frac{b}{c}$ = 0.9659, we see that cos 15° = $\frac{b}{c}$ = 0.9659.

The Cosine Ratio and Applications

In Figure 11.19, the cosine ratios become larger as θ decreases and become smaller as θ increases.

To understand why, consider the definition

$$\cos\theta = \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$$

and Figure 11.19.

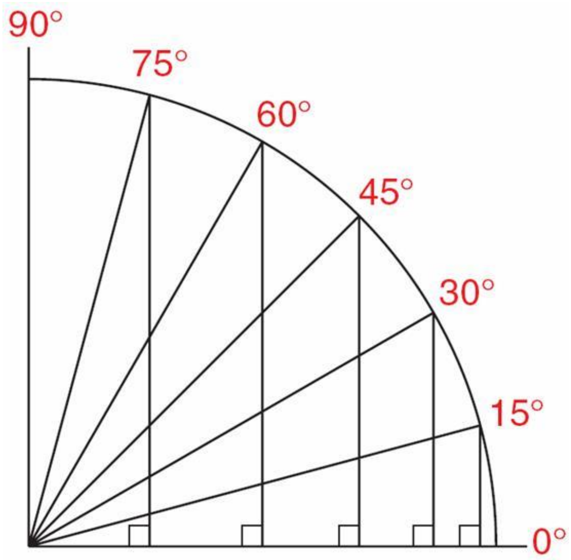

Figure 11.19

The Cosine Ratio and Applications

Recall that the symbol $\to$ is read "approaches."

As $\theta \to 0°$, length of adjacent leg $\to$ length of hypotenuse, and therefore cos 0° $\to$ 1.

Similarly, cos 90° $\to$ 0 because the adjacent leg grows smaller as $\theta \to 90°$.

Consequently, we have the following definition.

Definition

cos 0° = 1 and cos 90° = 0.

The Cosine Ratio and Applications

Use your calculator to verify the results found in Table 11.3.

TABLE 11.3

Cosine Ratios

θ	$\cos\theta$
0°	1.0000
15°	0.9659
30°	0.8660
45°	0.7071
60°	0.5000
75°	0.2588
90°	0.0000

The Cosine Ratio and Applications

In a right triangle, the cosine ratio can often be used to find either an unknown length or an unknown angle measure.

Whereas the sine ratio requires that we use *opposite* and *hypotenuse*, the cosine ratio requires that we use *adjacent* and *hypotenuse*.

The Cosine Ratio and Applications

An equation of the form $\sin \alpha = \frac{a}{c}$ or $\cos \alpha = \frac{b}{c}$ contains three variables; for the equation $\cos \alpha = \frac{b}{c}$, the variables are α, b, and c.

When the values of two of the variables are known, the value of the third variable can be determined; however, we must decide which trigonometric ratio is needed to solve the problem.

The Cosine Ratio and Applications

We now consider a statement that is called an **identity** because it is true for all angles; we refer to this statement as a theorem.

Theorem 11.2.1 (The Pythagorean Identity)

In any right triangle in which α is the measure of an acute angle,

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

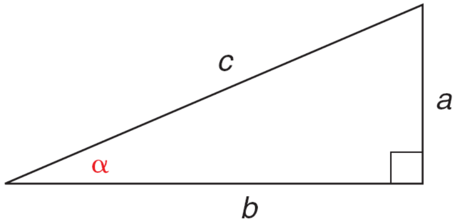

Figure 11.23

Note: $\sin^2 \alpha$ means $(\sin \alpha)^2$ and $\cos^2 \alpha$ means $(\cos \alpha)^2$.